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Chapter 26 Magnetism

"What is the fundamental hypothesis of science, the fundamental philosophy? [It is the following:] *the sole test of the validity of any idea is experiment.*"
Richard P. Feynman

26.1 The Force on a Charge in a Magnetic Field - The Definition of the Magnetic Field B

In addition to the existence of electric fields in nature, there are also magnetic fields. Most students have seen and played with a simple bar magnet, observing that the magnet attracts nails, paper clips, and the like. Some may have even placed the bar magnet under a piece of paper and sprinkled iron filings on the paper, observing the characteristic magnetic field of a bar magnet, figure 26.1. One

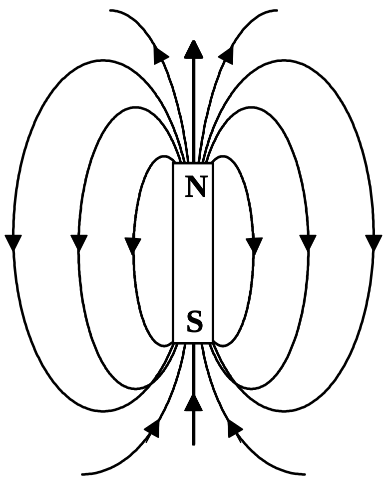


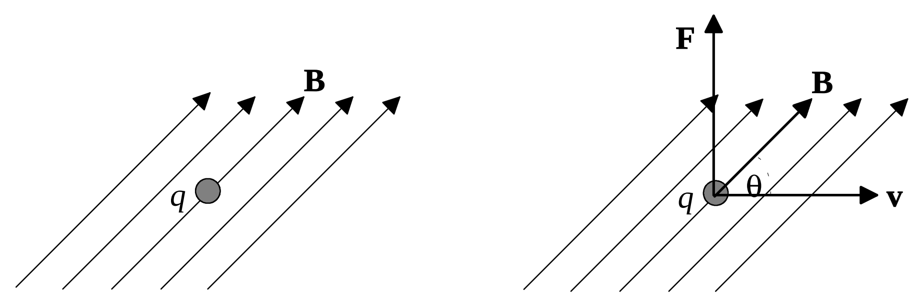
Figure 26.1 The magnetic field of a bar magnet.

end of the magnet is called a *north pole,* while the other end is called a *south pole.* The magnetic field is defined to emerge from the north pole of the magnet and enter at the south pole. A compass needle, a tiny bar magnet, placed in a magnetic field, aligns itself with the field. The designation of poles as north and south is arbitrary, just as electric charges are arbitrarily called positive and negative. Just as the combination of a positive and a negative electric charge is called an electric dipole, a bar magnet, consisting as it does of a north and a south magnetic pole, is sometimes called a *magnetic dipole.* The force between magnets is similar to the force between electric charges and can be stated as *the **fundamental principle of magnetostatics:** like magnetic poles repel, while unlike magnetic poles attract.* The earth has a magnetic field and when the north pole of a compass needle points in a northerly direction on the surface of the earth, it is really being attracted toward a south magnetic pole, because unlike poles attract. Hence, what is usually called the north magnetic pole of the earth is really a south pole. (Because compass needles always point toward that pole, it is sometimes erroneously called the north magnetic pole. Thus a pilot or navigator would refer to it as magnetic north.) This south magnetic pole is displaced about 1300 miles from the north geographic pole of the earth. Similarly, when the south pole of a compass needle points in a southerly direction on the surface of the earth, it is really being attracted toward a north magnetic pole. The north magnetic pole is displaced about 1200 miles from the south geographic pole of the earth. We will use the symbol **B** to designate the magnetic field. Later in this chapter we will see how magnetic fields are generated, but for now let us accept the fact that magnetic fields do indeed exist.

Recall from chapter 21 that we determined the existence of an electric field and its strength by the effect it produced on a small positive test charge q_0 placed in the region where we assumed the field to exist. If the test charge experienced a force, we said that the charge was in an electric field, and defined the electric field as

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} \tag{21.1}$$

It is desirable to define the magnetic field in a similar way. A positive charge q is placed at rest in a uniform magnetic field **B** as shown in figure 26.2(a). But to our surprise, nothing happens to the charge. No force is observed to act on the



(a) No force on charge at rest (b) Force on moving charge

Figure 26.2 A charge in a magnetic field.

charge. The experiment is repeated, but now the charge is fired into the magnetic field with a velocity **v**, figure 26.2(b). It is now observed that a force does indeed act on the charge. The force, however, acts at an angle of 90^0 to the plane determined by the velocity vector **v**, and the magnetic field vector **B**. In fact, *the force acting on the moving charge is given by the cross product of* **v** *and* **B** *as*

$$\boldsymbol{F} = q\boldsymbol{v} \times \boldsymbol{B} \tag{26.1}$$

The magnitude of the force is determined from the definition of the cross product, equation 3.53, and is given by

$$F = qvB\sin\theta \tag{26.2}$$

The angle θ is the angle between the velocity vector **v** and the magnetic field vector **B**. If the angle θ between **v** and **B** is 90^0 then the magnitude of the force simplifies to

$$F = qv_{\perp}B \tag{26.3}$$

This result is now used to define the magnitude of the magnetic field B as

$$B = \frac{F}{qv_{\perp}} \tag{26.4}$$

This definition is now similar to the definition of the electric field. *The magnetic field B, called the magnetic induction or the magnetic flux density, is defined as the force per unit charge, per unit velocity, provided that v is perpendicular to B*. The SI unit for the magnetic induction is defined from equation 26.4 to be a tesla, named after Nikola Tesla (1856-1943), where

$$1 \text{ tesla} = 1 \frac{\text{newton}}{\text{coulomb m/s}}$$

This will be abbreviated as

$$1 \text{ T} = 1 \frac{\text{N}}{\text{C m/s}}$$

The relation of the Tesla with other equivalent units for the magnetic field are

$$\text{tesla} = \frac{\text{N}}{\text{A m}} = \frac{\text{weber}}{\text{m}^2} = 10^4 \text{ gauss}$$

The gauss is a cgs unit that is still used because of its convenient size. To give you an idea of the size of the tesla, the earth's magnetic field is about 1/20000 tesla.

From the definition of the cross product it is obvious that if the charge has a velocity which is parallel to the magnetic field, then the angle θ will be zero and hence

$$F = qvB \sin 0^0 = 0 \quad (\text{for } \mathbf{v} \mid\mid \mathbf{B}) \tag{26.5}$$

Of course, if the velocity of the charge is zero then the force will also be zero. The magnetic field manifests itself to a charge only when the charge is in motion with respect to the field. The maximum force occurs when **v** is at an angle of 90^0 to **B**, as shown in equation 26.3. It should also be stated that equation 26.1 was defined with q being a positive charge. If the charge in motion is a negative particle, such as an electron, q will be negative and the force on the negative particle will be in the opposite direction to the force on the positive particle.

Example 26.1

The force on a positive charge in a magnetic field. A proton is fired into a uniform magnetic field **B** of magnitude 0.500 T, at a speed of 300 m/s at an angle of 30.0^0 to **B**. Find the force and the acceleration of the proton.

Solution

The magnitude of the force acting on the proton is found from equation 26.2 as

$$F = qvB\sin\theta = (1.60 \times 10^{-19}\text{ C})(300\text{ m/s})(0.500\text{ T})\text{ Sin}30.0^0$$
$$F = (1.2 \times 10^{-17}\text{ C (m/s) T})\frac{(\text{N/(C m/s)})}{(\text{ T })}$$
$$F = 1.20 \times 10^{-17}\text{ N}$$

Note how the conversion factor for a tesla was used to make the force come out in the unit of newtons. The direction of the force is perpendicular to the plane of **v** and **B** as shown in figure 26.2(b). The acceleration of the proton is found from Newton's second law as

$$a = \frac{F}{m_p} = \frac{1.20 \times 10^{-17}\text{ N}}{1.67 \times 10^{-27}\text{ kg}} = 7.19 \times 10^9\text{ m/s}^2$$

As long as the particle stays within the magnetic field, at the same angle θ, the magnitude of the force and the magnitude of the acceleration is a constant. Since the magnitude of the acceleration is a constant, the kinematic equations from college physics can be used to find the position of the particle at any time.

To go to this Interactive Example click on this sentence.

Example 26.2

A force on an electron in a magnetic field. An electron is fired into a uniform magnetic field with a velocity **v,** as shown in figure 26.3. Find the direction of the magnetic force.

Solution

The direction of the force is perpendicular to the plane of **v** and **B.** However, since the magnitude of the force is given by

$$F = qvB\sin\theta$$

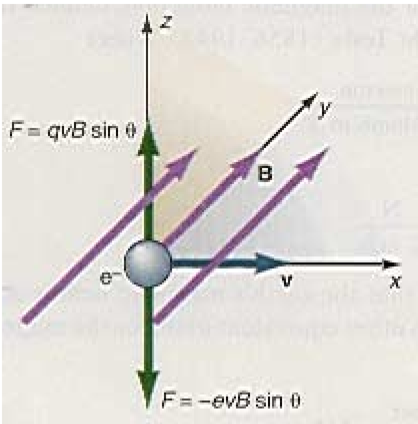


Figure 26.3 Force on a negative charge in a magnetic field.

and the electron has a negative charge, $q = -e$, the force on the electron is

$$F = -evB \sin\theta$$

As we can see from figure 26.3, the force **F** points in the negative direction and points downward. (Recall that the direction of the force on a charge in a magnetic field is found by the right-hand rule. That is, point the fingers of your right hand in the direction of the velocity vector **v** with your palm facing the magnetic field vector **B.** Rotate your right hand from **v** to **B** and your thumb will point in the direction of the force vector, in this case upward.) But since the force is minus e times $vB \sin\theta$, for the negative charge, the force **F** points downward.

Because a charge can experience a force in either an electric or a magnetic field, the total force on a particle in an electromagnetic field is the superposition of both forces, that is,

$$\mathbf{F} = \mathbf{F_e} + \mathbf{F_m}$$

or

$$\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B} \qquad (26.6)$$

Equation 26.6 is known as the **Lorentz force** law.

An interesting result occurs if we look at the work done on the particle in such a field. The work is defined as

$$W = \mathbf{F} \bullet \mathbf{x}$$

But the total force acting on the charge was given by equation 26.6. Hence the work done by the Lorentz force is

$$W = q\mathbf{E} \bullet \mathbf{x} + q(\mathbf{v} \times \mathbf{B}) \bullet \mathbf{x} \qquad (26.7)$$

Let us consider the second term $q(\mathbf{v} \times \mathbf{B})\bullet\mathbf{x}$. Since $\mathbf{x}$ is the displacement of the moving charge from its initial position to its next position, it is always in the direction of $\mathbf{v}$ at any point. Since the magnetic force $q(\mathbf{v} \times \mathbf{B})$ is perpendicular to $\mathbf{v}$ it is also perpendicular to the displacement vector $\mathbf{x}$. Hence the angle between the magnetic force and the displacement is 90^0, and the dot product term $(\mathbf{v} \times \mathbf{B})\bullet\mathbf{x}$ is equal to zero and thus the work done by the magnetic force is zero. That is,

$$W_{\mathbf{m}} = q(\mathbf{v} \times \mathbf{B})\bullet\mathbf{x} = 0$$

Therefore, equation 26.7 reduces to

$$W = q\mathbf{E}\bullet\mathbf{x}$$

or

$$W = qEx \cos \theta_1 \qquad (26.8)$$

Equation 26.8 says that only the work done on the particle by the electric field can change the energy, and hence the speed, of the particle. The magnetic field can only change the direction of the velocity vector, but not its speed. Therefore, a particle moving in a magnetic field only, always moves at a constant speed.

If a charged particle q enters a uniform magnetic field $\mathbf{B}$ with a velocity $\mathbf{v}$**,** as shown in figure 26.4, the magnetic force $\mathbf{F}$ acts on the particle, deflecting it from its

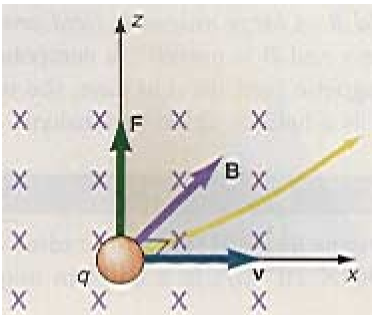


Figure 26.4 Deflecting a charged particle in a magnetic field.

straight line motion. The particle follows the curved path until it emerges from the magnetic field. It then moves in a straight line at a constant velocity. Uniform magnetic fields are sometimes used in this way to deflect charged particles in a cathode ray oscilloscope. (Note that in figure 26.4 the symbol $\times$ represents the tail of the arrow of the vector $\mathbf{B}$**,** indicating that $\mathbf{B}$ is *into the page.* If $\mathbf{B}$ were *out of the page*, it would be represented by dots ($\cdot$), indicating that the tip of the arrow of the vector $\mathbf{B}$ is coming out of the page.)

If the uniform magnetic field covers a large enough area, and the velocity vector makes an angle of 90^0 with the magnetic field, the particle stays in the magnetic field and moves in a circle. Consider the particle, of charge q, entering the uniform magnetic field $\mathbf{B}$ in figure 26.5. The magnetic field is everywhere into the paper and is perpendicular to $\mathbf{v}$**.** When the charge is at position 1, moving at a velocity $\mathbf{v}_1$, it experiences a force $\mathbf{F}_1$, which acts upward. (Recall that the direction of

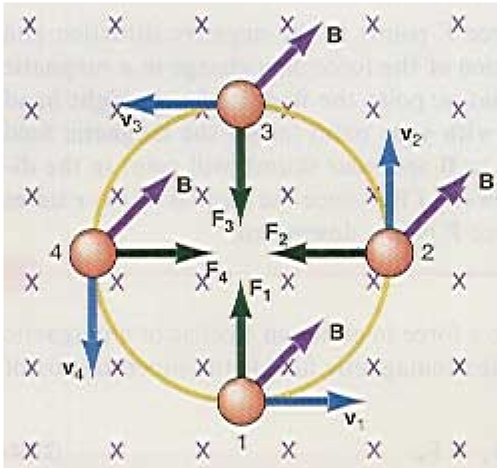


Figure 26.5 Deflecting a charged particle into a circular path.

the force on a charge in a magnetic field is found by the right-hand rule. Point the fingers of your right hand in the direction of the velocity vector **v** with your palm facing the magnetic field vector, **B.** Then rotate your right hand from **v** to **B** and your thumb will point in the direction of the force vector.) This force deflects the charge from its straight line motion and it follows the yellow path. All along that path a force is always acting perpendicular to the path. When the charge is at position 2, the force on it is now toward the left, again deflecting the direction of motion of the charge as shown. At position 3, the force acts downward, whereas at position 4 it acts toward the right. The particle is again deflected until it returns to its initial position 1. At every point on the trajectory of the particle, the force always acts perpendicular to the velocity vector **v.** Hence in this case, the magnetic force $F_{\mathbf{m}} = qvB \sin \theta$ is a centripetal force. The particle moves in a circular orbit in the uniform magnetic field at constant speed *v*. (The speed *v* remains constant because only an electric field can change the particle's speed.)

The magnetic force supplies the necessary centripetal force for the particle to move in the circular path, that is,

$$F_{\mathbf{c}} = F_{\mathbf{m}}$$

In this case **v** and **B** are perpendicular and hence $\theta = 90^0$, the $\sin 90^0 = 1$, and

$$F_{\mathbf{m}} = qvB \tag{26.9}$$

Equating the centripetal force, equation 6.14, to the magnetic force, equation 26.9, gives

$$\frac{mv^2}{r} = qvB \tag{26.10}$$

Solving equation 26.10 for *r*, the radius of the orbit, gives

$$r = \frac{mv}{qB} \tag{26.11}$$

Since mv *is the momentum of the charged particle, the radius of the orbit is directly proportional to the momentum of the particle. The larger the momentum, the larger the circular orbit. The orbital radius is inversely proportional to the charge of the particle* q *and the magnetic field* B. *A large magnetic field produces a small circular orbit.* If the angle θ between **v** and **B** is not 90^0, a component of the velocity will be in the direction of the magnetic field. In that case, the motion does not remain in a plane and the trajectory is a helix in three dimensions.

Example 26.3

The radius of the orbit of an electron in a magnetic field. Find the radius of the orbit of an electron moving at a speed of 2.00×10^7 m/s in a uniform magnetic field of 1.20×10^{-3} T.

Solution

The radius of the orbit, found from equation 26.11, is

$$r = \frac{mv}{qB} = \frac{(9.11 \times 10^{-31}\ \text{kg})(2.00 \times 10^7\ \text{m/s})}{(1.61 \times 10^{-19}\ \text{C})(1.20 \times 10^{-3}\ \text{T})}$$

$$= \left(9.43 \times 10^{-2} \frac{\text{kg m/s}}{\text{C T}}\right)\left(\frac{\text{T}}{\frac{\text{N}}{\text{C m/s}}}\right)\left(\frac{\text{N}}{\text{kg m/s}^2}\right)$$

$$= 9.43 \times 10^{-2}\ \text{m}$$

$$= 9.43\ \text{cm}$$

Note how the conversion factors were used to give the radius in the correct units.

To go to this Interactive Example click on this sentence.

If an electric and a magnetic field are superimposed at right angles to each other, as shown in figure 26.6, the arrangement acts as a **velocity selector.** The force on charge q is given by the Lorentz force, equation 26.6, as

$$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$$

The magnetic force, $q\mathbf{v} \times \mathbf{B}$, acts upward while the electric force $\mathbf{F_e} = q\mathbf{E}$ acts downward. If the electric force is exactly equal in magnitude to the magnetic force, the forces cancel each other out and the net force **F** on the charged particle is zero.

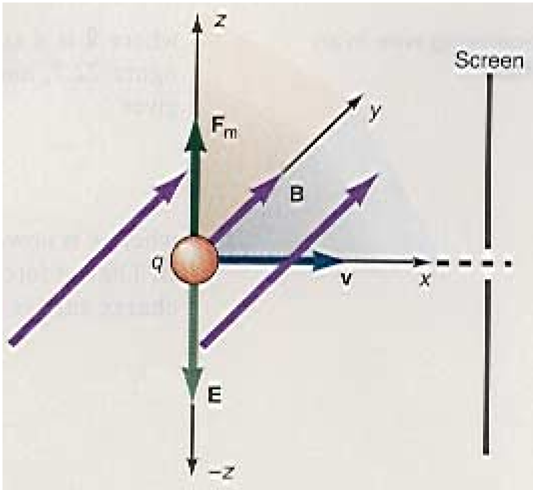


Figure 26.6 A velocity selector.

The particle is not deviated from its straight line motion. The requirement for the velocity to be undeviated as it moves through the combined fields, obtained from equation 26.6, is

$$0 = -qE + qvB \sin 90^0$$
$$qE = qvB$$

and

$$v = \frac{E}{B} \tag{26.12}$$

Therefore, particles whose speed v is given by equation 26.12 pass through the combined fields undeflected and pass through the slit in the screen in figure 26.6. All particles with a different velocity have a net force acting on them and are deflected from their straight line motion and do not pass through the slit in the screen.

Example 26.4

A velocity selector. Alpha particles ranging in speeds from 1000 m/s to 2000 m/s enter an electromagnetic field, as in figure 26.6, where the electric intensity is 300 V/m and the magnetic induction is 0.200 T. Which particles will move undeviated through the field?

Solution

The alpha particles moving at a speed given by equation 26.12 are selected to pass straight through the field, that is,

$$v = \frac{E}{B}$$

$$= \frac{300 \text{ V/m}}{0.200 \text{ T}}$$
$$= 1500 \text{ m/s}$$

Only particles moving at this speed move straight through the electromagnetic field; all others are deflected.

To go to this Interactive Example click on this sentence.

26.2 Force on a Current-Carrying Conductor in an External Magnetic Field

If a wire carrying a current I is placed in an external magnetic field **B** as shown in figure 26.7, a force will be found to act on the wire. The explanation for this force

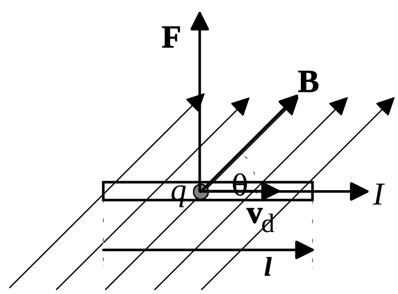


Figure 26.7 Force on a current-carrying wire in an external magnetic field

can be found in the magnetic force acting on a charged particle in a magnetic field. If the wire is carrying a current, then there are charges in motion within the wire. These charges will be moving with a drift velocity $\mathbf{v}_d$ in the direction of the current flow. Any one of these charges q will experience the force

$$\mathbf{F}_q = q\mathbf{v}_d \times \mathbf{B} \tag{26.13}$$

This force on an individual charge will cause the charge to interact with the lattice structure of the wire, exerting a force on the lattice and hence the wire itself. The drift velocity of the moving charge can be written as

$$\mathbf{v}_d = \frac{\boldsymbol{l}}{t} \tag{26.14}$$

where $\boldsymbol{l}$ is a small length of the wire in the direction of the current flow and is shown in figure 26.14, and t is the time. Replacing this drift velocity in equation 26.13 gives

$$\boldsymbol{F}_q = q\left(\frac{\boldsymbol{l}}{t}\right) \times \boldsymbol{B} = \left(\frac{q}{t}\right)\boldsymbol{l} \times \boldsymbol{B} \qquad (26.15)$$

The net force on the wire is the sum of the individual forces associated with each charge carrier, i.e.,

$$\boldsymbol{F} = \Sigma_q \boldsymbol{F}_q = \Sigma_q \left(\frac{q}{t}\right)\boldsymbol{l} \times \boldsymbol{B}$$

But Σ_q (q/t) is equal to all the charges passing through a plane of the wire per unit time and is defined to be the current in the circuit, I. Hence *a wire carrying a current I in any external magnetic field* **B**, *will experience a force given by*

$$\boldsymbol{F} = I\,\boldsymbol{l} \times \boldsymbol{B} \qquad (26.16)$$

The force is again given by a cross product term, and the direction of the force is found from $\boldsymbol{l} \times \mathbf{B}$. With $\boldsymbol{l}$ in the direction of the current and **B** pointing into the page in figure 26.7, $\boldsymbol{l} \times \mathbf{B}$ is a vector that points upward. If the direction of the current flow is reversed, $\boldsymbol{l}$ would be reversed and $\boldsymbol{l} \times \mathbf{B}$ would then point downward. *The magnitude of the force is determined from equation 26.16 as*

$$F = IlB \sin\theta \qquad (26.17)$$

where θ is the angle between $\boldsymbol{l}$ and **B**. Solving equation 26.17 for B gives another set of units for the magnetic induction B, namely

$$B = \frac{F}{Il}$$

Thus,

$$1 \text{ tesla} = \frac{\text{newton}}{\text{ampere meter}}$$

This will be abbreviated as

$$1 \text{ T} = 1 \frac{\text{N}}{\text{A m}}$$

Example 26.5

The force on current-carrying wire in a magnetic field. A 20.0 cm wire carrying a current of 10.0 A is placed in a uniform magnetic field of 0.300 T as shown in figure 26.7. If the wire makes an angle of 40.0^0 with the vector **B**, find the direction and magnitude of the force on the wire.

Solution

The direction of the force is found from equation 26.16 as

$$\mathbf{F} = I\boldsymbol{l} \times \mathbf{B}$$

In rotating the vector $\boldsymbol{l}$ toward the vector $\mathbf{B}$ in the cross product, the thumb points upward, indicating that the direction of the force is also upward. The magnitude of the force is found from equation 26.17 as

$$F = IlB \sin\theta$$
$$F = (10.0 \text{ A})(0.200 \text{ m})(0.300 \text{ T}) \sin 40.0^0$$
$$F = (0.386 \text{ A m T})\left(\frac{\text{N/(A m)}}{\text{T}}\right)$$
$$F = 0.386 \text{ N}$$

To go to this Interactive Example click on this sentence.

26.3 Force on a Semicircular Wire Carrying a Current in an External Magnetic Field

In section 26.2 we found the force on a straight piece of wire carrying a current in an external magnetic field is given by

$$\boldsymbol{F} = I\,\boldsymbol{l} \times \boldsymbol{B} \tag{26.16}$$

But what if the wire is not a straight wire, what is the force on the wire then? If the wire is not straight, we divide the wire into small little elements of length $d\boldsymbol{l}$, each in the direction of the current flow, and each length $d\boldsymbol{l}$ experiences the force $d\mathbf{F}$ given by

$$d\boldsymbol{F} = I\,d\boldsymbol{l} \times \boldsymbol{B} \tag{26.18}$$

The total force on the wire will now be just the sum, or integral, of all these $d\mathbf{F}$'s. That is,

$$\boldsymbol{F} = \int d\boldsymbol{F} = \int I\,d\boldsymbol{l} \times \boldsymbol{B} \tag{26.19}$$

As an example of the application of equation 26.19 let us find the force on a semicircular piece of wire carrying a current as shown in figure 26.8. Consider the small piece of wire $d\boldsymbol{l}$ that is shown. The magnetic field $\mathbf{B}$ points into the page and $d\boldsymbol{l} \times \mathbf{B}$ points down along the radius of the semicircle to the center of the circle. The

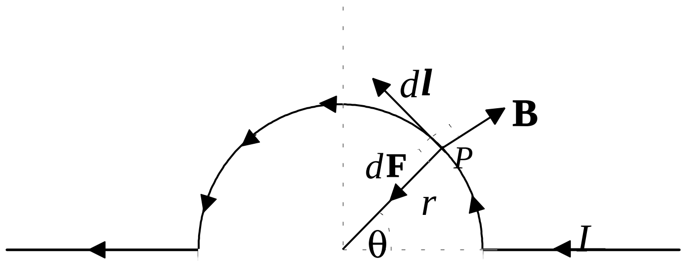


Figure 26.8 Force on a semicircular portion of wire.

angle between **B** and $d\boldsymbol{l}$ is 90^0, therefore the element of force $d\mathbf{F}$ associated with this small element $d\boldsymbol{l}$ of wire is

$$dF = I\ dl\ B \sin 90^o\ (-r_o) = I\ dl\ B\ (-r_o) \tag{26.20}$$

$\mathbf{r}_o$ is a unit vector that points from the origin to the point P, and $d\mathbf{F}$ points in the opposite direction or $-\ \mathbf{r}_o$. The total force on the semicircular wire is the sum of all these $d\mathbf{F}$'s or

$$\boldsymbol{F} = \int d\boldsymbol{F} = \int_0^\pi I\ dl\ B(-\boldsymbol{r}_o) \tag{26.21}$$

But the unit vector $\mathbf{r}_o$ can be written in terms of the unit vectors $\mathbf{i}$ and $\mathbf{j}$ as

$$\boldsymbol{r}_o = \boldsymbol{i}\cos\theta + \boldsymbol{j}\sin\theta \tag{26.22}$$

Replacing equation 26.22 into equation 26.21 we get

$$\boldsymbol{F} = \int_0^\pi I\ dl\ B(-\boldsymbol{r}_o) = \int_0^\pi I\ B\ dl\ [-(\boldsymbol{i}\cos\theta + \boldsymbol{j}\sin\theta)]$$
$$\boldsymbol{F} = -\int_0^\pi I\ B\ dl\ \boldsymbol{i}\cos\theta - \int_0^\pi IB\ dl\ \boldsymbol{j}\sin\theta \tag{26.23}$$

But dl and θ are not independent and are related by

$$dl = r\ d\theta \tag{26.24}$$

Therefore equation 26.23 becomes

$$\boldsymbol{F} = -\int_0^\pi I\ B\ rd\theta\ \boldsymbol{i}\cos\theta - \int_0^\pi IB\ rd\theta\ \boldsymbol{j}\sin\theta$$

The current I in the circuit is a constant, as well as the magnetic field B, and the radius of the semicircle r, and they can each be removed from under the integral sign to yield

$$\boldsymbol{F} = -IBr\int_0^\pi \cos\theta d\theta\ \boldsymbol{i} - IBr\int_0^\pi \sin\theta d\theta\ \boldsymbol{j}$$

Performing the integrations

$$F = -IBr \sin\theta \,|_0^\pi\, \boldsymbol{i} - IBr(-\cos\theta)\,|_0^\pi\, \boldsymbol{j}$$
$$F = -IBr[\sin\pi - \sin 0]\, \boldsymbol{i} + IBr[\cos\pi - \cos 0]\, \boldsymbol{j}$$
$$F = -IBr[0 - 0]\, \boldsymbol{i} + IBr[-1 - 1]\, \boldsymbol{j} = 0\, \boldsymbol{i} - 2IBr\, \boldsymbol{j}$$

and finally

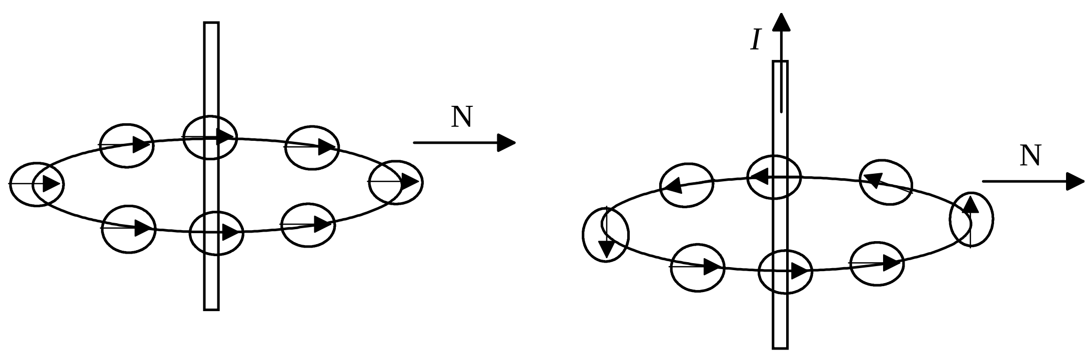
$$\boldsymbol{F} = -2rIB\, \boldsymbol{j} \tag{26.25}$$

Equation 26.25 gives the force **F** *that will act on the semicircular portion of the wire carrying a current I in a magnetic field B.* Notice that the force is completely in the negative **j** direction, or downward. The two components in the *x*-direction cancel each other out.

Although this problem was solved for a semicircular portion of a current-carrying wire, the procedure is essentially the same for any shape of wire. That is, *d***F** is given by equation 26.18 and the total force will be the sum or integral of all these *d***F**'s in equation 26.19.

26.4 Generation of a Magnetic Field

Although magnets had been known for hundreds of years, it was not until 1820, that Hans Christian Oersted (1777 - 1851) discovered a relation between electric current and magnetic fields. If a series of compasses are placed around a wire that is not carrying a current, all the compasses will point toward the north, the direction of the earth's magnetic field, as shown in figure 26.9(a). If, however, a current *I* is sent through the wire, the compass needles will no longer point to the



(a) No current in wire (b) Current in wire

Figure 26.9 The creation of a magnetic field by an electric current

north. Instead they point in a direction which is everywhere tangential to a circle drawn around the wire, passing through each compass, as shown in figure 26.9(b). Because a compass always aligns itself in the direction of a magnetic field, the current in the wire has created a circular magnetic field directed counterclockwise around the wire. If the direction of the current is reversed, the direction of the magnetic field will also be reversed and the compasses will point in a clockwise direction. *The direction of the magnetic field around a long straight wire carrying a current, I, is easily determined by the so called "Right Hand Wire Rule": Grasp the wire with the right hand, with the thumb in the direction of the current flow, the fingers will curl around the wire in the direction of the magnetic field.*

This observation that electric currents can create a magnetic field was responsible for linking the then two independent sciences of electricity and magnetism into the one unified science of electromagnetism. In fact, it will be shown later that all magnetic fields are caused by the flow of electric charge.

26.5 The Biot-Savart Law

*The Biot-Savart law relates the amount of magnetic field d**B** at the position* **r** *produced by a small element d**l**, of a wire carrying a current I and is given by*

$$d\boldsymbol{B} = \frac{\mu I}{4\pi}\frac{d\boldsymbol{l}\times\boldsymbol{r}}{r^3} \tag{26.26}$$

and is shown in figure 26.10(a). μ is a constant called the permeability of the

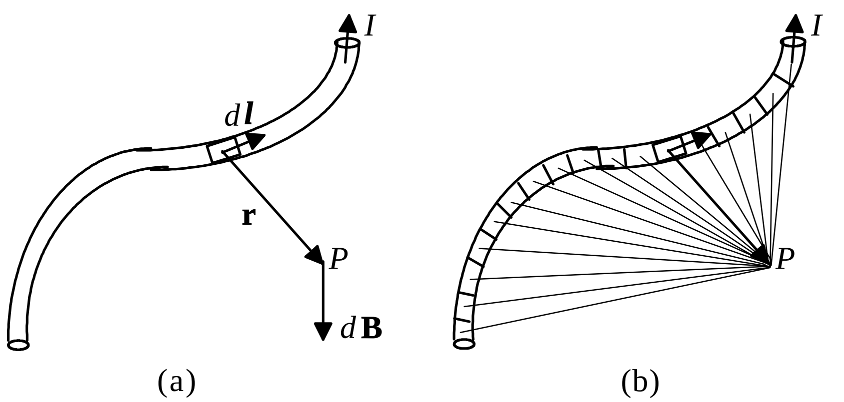


Figure 26.10 The magnetic field produced by a current element.

medium. In a vacuum or air, it is called the permeability of free space, and is denoted by μ_o, where

$$\mu_o = 4\pi \times 10^{-7}\ \frac{\text{T m}}{\text{A}}$$

The cross product term $d\boldsymbol{l} \times \mathbf{r}$ immediately determines the direction of $d\mathbf{B}$, and is shown in figure 26.10(a). The Biot-Savart law says that the small current element $d\boldsymbol{l}$ produces a small amount of magnetic field $d\mathbf{B}$ at the point *P*. But the entire length of wire can be cut up into many $d\boldsymbol{l}$'s, and each will contribute to the total magnetic field at *P*. This is shown in figure 26.10(b). *Therefore, the total magnetic field at point P is the vector sum, and hence, integral of all the d**B**'s associated with each current element, i.e.,*

$$\boldsymbol{B} = \int d\boldsymbol{B} \tag{26.27}$$

The computation of the magnetic field from equation 26.27 can be quite complicated for most problems because of the vector integration. However, there are some problems that can be easily solved by the Biot-Savart law, and we will solve some in the next sections.

26.6 The Magnetic Field at the Center of a Circular Current Loop

To determine the magnetic field at the center of a circular current loop, figure 26.11, the Biot-Savart law is used. A small element of the wire $d\boldsymbol{l}$ produces an element of magnetic field $d\mathbf{B}$ at the center of the wire given by equation 26.26 as

$$d\boldsymbol{B} = \frac{\mu_o I}{4\pi}\frac{d\boldsymbol{l}\times\boldsymbol{r}}{r^3}$$

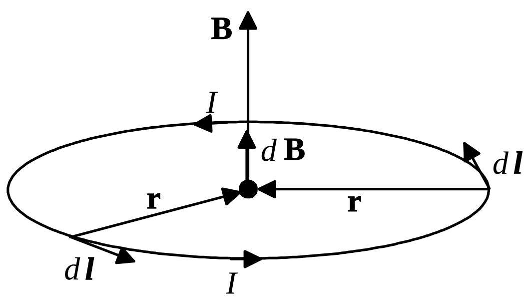


Figure 26.11 The magnetic field at the center of a circular current loop

From the nature of the vector cross product, $d\boldsymbol{l} \times \mathbf{r}$, and hence $d\mathbf{B}$ points upward at the center of the circle for every current element, as seen in figure 26.11. The total magnetic field $\mathbf{B}$ which is the sum of all the $d\mathbf{B}$'s, must also point upward at the center of the loop. Therefore, the magnetic field at the center of the current loop is perpendicular to the plane formed by the loop, and points upward. Since the direction of the vector $\mathbf{B}$ is now known, equation 26.27 can be reduced to the scalar form

$$B = \int dB \tag{26.28}$$

The magnitude of dB is found from equation 26.26 to be

$$dB = \frac{\mu_o I}{4\pi}\frac{dl r \sin\theta}{r^3} \tag{26.29}$$

Since $d\boldsymbol{l}$ is perpendicular to $\mathbf{r}$, the angle θ is equal to 90^0, and the sine of 90^0 is equal to 1. Therefore, dB becomes

$$dB = \frac{\mu_o I dl}{4\pi r^2} \tag{26.30}$$

Replacing equation 26.30 into equation 26.28 gives the magnitude of the magnetic induction B as

$$B = \int dB = \int \frac{\mu_o I dl}{4\pi r^2} \tag{26.31}$$

Because the loop is a circle of constant radius r and $\mu_o I/4\pi$, is a constant, these terms will be in every term of the summation and can be factored out of the integral to yield

$$B = \frac{\mu_o I}{4\pi r^2} \int dl \tag{26.32}$$

But the summation of all the dl's is simply the circumference of the wire, i.e.,

$$\int dl = 2\pi r \tag{26.33}$$

Therefore, equation 26.32 becomes

$$B = \frac{\mu_o I}{4\pi r^2}(2\pi r) \tag{26.34}$$

Canceling like terms, this becomes

$$B = \frac{\mu_o I}{2r} \tag{26.35}$$

Equation 26.35 gives the magnetic field at the center of a circular current loop. Notice that the magnetic field at the center of the circular current loop is directly proportional to the current I - the larger the current, the larger the magnetic field; and inversely proportional to the radius r of the loop - the larger the radius, the smaller the magnetic field. If there are N turns of wire constituting the loop, the magnetic field at the center is

$$B = \frac{\mu_o N I}{2r} \tag{26.36}$$

The magnetic field found in this way is the magnetic field at the center of the current loop. The magnetic field all around the loop is shown in figure 26.12. Note that it looks something like the magnetic field of a bar magnet, where the top of the loop would be the north pole.

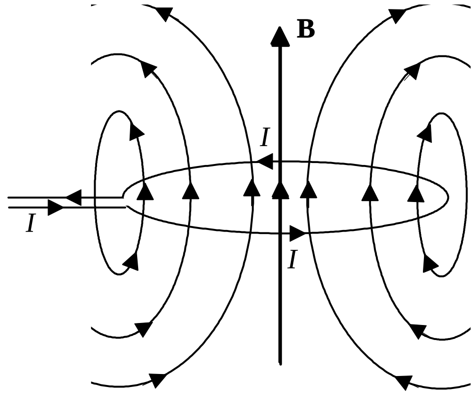


Figure 26.12 The magnetic field of a current loop.

Example 26.6

The magnetic field at the center of a circular current loop. Find the magnetic field at the center of a circular current loop of 0.500 m radius, carrying a current of 7.00 A.

Solution

The magnetic field at the center of the loop, found from equation 26.35, is

$$B = \frac{\mu_o I}{2r} = \frac{(4\pi \times 10^{-7}\text{T m/A})(7.00\text{ A})}{2(0.500\text{ m})}$$
$$B = 8.80 \times 10^{-6}\text{ T}$$

To go to this Interactive Example click on this sentence.

Example 26.7

A circular loop of N turns. Find the magnetic field at the center of a circular current loop of 10 turns, with a radius of 5.00 cm carrying a current of 10.0 A.

Solution

The magnetic field is found from equation 26.36 to be

$$B = \frac{\mu_o NI}{2r} = \frac{(4\pi \times 10^{-7}\text{T m/A})(10)(10.0\text{ A})}{2(0.050\text{ m})}$$
$$B = 1.26 \times 10^{-3}\text{ T}$$

To go to this Interactive Example click on this sentence.

26.7 Magnetic Field on Axis for a Circular Current Loop

Let us now consider a case a little more general than the one we solved in section 26.6, by finding the magnetic field **B** at a point *P* on the *z*-axis of a current loop that lies in the *x*, *y*-plane, figure 26.13. We consider a small element $d\boldsymbol{l}$ of the current

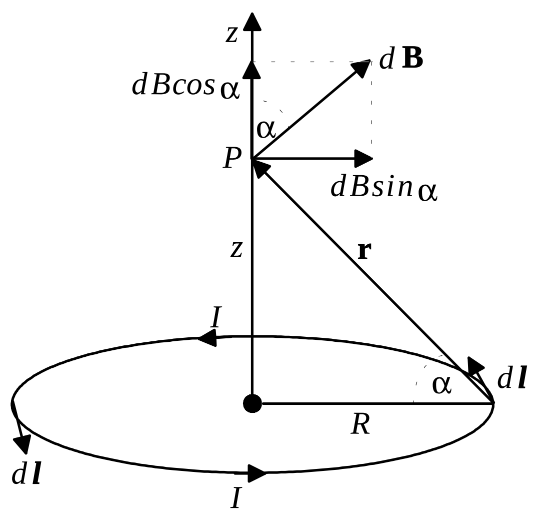


Figure 26.13 The magnetic field on axis for a circular current loop.

carrying wire and use the Biot-Savart law to find the element of the magnetic field $d\mathbf{B}$ associated with this element of wire $d\boldsymbol{l}$. That is,

$$d\boldsymbol{B} = \frac{\mu_o I}{4\pi}\frac{d\boldsymbol{l} \times \boldsymbol{r}}{r^3} \tag{26.26}$$

The total magnetic field at the point *P* is the sum or integral of all the $d\mathbf{B}$'s caused by all the $d\boldsymbol{l}$'s as we go around the circle, that is

$$\boldsymbol{B} = \int d\boldsymbol{B} \tag{26.27}$$

The integration in equation 26.27 is a vector integration. We simplify the problem by noting that the vector $d\mathbf{B}$ has the components $dB \sin\alpha$ and $dB \cos\alpha$. As you can see in figure 26.13, the term $dB \sin\alpha$ points to the right in the positive *x*-direction. But diametrically opposite to the element $d\boldsymbol{l}$ considered, is another $d\boldsymbol{l}$ and there is a corresponding $d\mathbf{B}$ associated with it that will also have a component $dB \sin\alpha$, but this component points to the left in the negative *x*-direction. For each $+dB \sin\alpha$ associated with an element $d\boldsymbol{l}$ on one side of the wire there will be a corresponding

$-dB$ sinα on the opposite side and the sum of all these dB sinα terms will be zero. This is equivalent to saying that the sum or integral of all these components will be zero. That is,

$$\int dB \sin \alpha = 0$$

But notice that the terms dB cosα always point in the positive z-direction, and it is the sum or integral of these components that will give us the magnetic field at the point P. Therefore, the magnetic field **B** is found from

$$B = \int dB \cos \alpha \tag{26.37}$$

Upon substituting for dB we get

$$B = \int \frac{\mu_o I}{4\pi} \frac{dl \sin 90^0}{r^2} \cos \alpha$$

$$B = \frac{\mu_o I}{4\pi} \int \frac{\cos \alpha}{r^2} dl \tag{26.38}$$

But the variables r and α are not independent of each other and, as can be seen from the geometry of figure 26.13, the relations are

$$r = \sqrt{R^2 + z^2}$$

and

$$\cos \alpha = \frac{R}{\sqrt{R^2 + z^2}}$$

Replacing these into equations 26.38 yields

$$B = \frac{\mu_o I}{4\pi} \int \frac{\cos \alpha}{r^2} dl = \frac{\mu_o I}{4\pi} \int \frac{\frac{R}{\sqrt{R^2+z^2}}}{R^2 + z^2} dl = \frac{\mu_o IR}{4\pi} \int \frac{dl}{(R^2 + z^2)^{\frac{3}{2}}}$$

But as you can see in figure 26.13 for the point P, the values of R and z are constant and hence can be taken out of the integral sign to yield

$$B = \frac{\mu_o IR}{4\pi(R^2 + z^2)^{\frac{3}{2}}} \int dl \tag{26.39}$$

Thus the only integration is the sum of all the dl's. But the sum of all the dl's is just the circumference of the circle. Therefore,

$$\int dl = 2\pi R \tag{26.40}$$

Substituting equation 26.40 into 26.39 gives

$$B = \frac{\mu_o IR}{4\pi(R^2 + z^2)^{\frac{3}{2}}} 2\pi R$$

$$B = \frac{\mu_o IR^2}{2(R^2 + z^2)^{\frac{3}{2}}} \qquad (26.41)$$

Equations 26.41 gives the magnetic field B at the point P located on the z-axis of a circular current loop of radius R carrying a current I.

Example 26.8

The magnetic field B at the center of a circular current loop. From the solution for the magnetic field on axis for a circular current loop, find the magnetic field B at the center of the circular current loop.

Solution

The magnetic field at the center of the circular current loop is found from equation 26.41 by letting $z = 0$. Therefore,

$$B = \frac{\mu_o IR^2}{2(R^2 + z^2)^{\frac{3}{2}}} = \frac{\mu_o IR^2}{2(R^2 + 0)^{\frac{3}{2}}} = \frac{\mu_o IR^2}{2(R^2)^{\frac{3}{2}}} = \frac{\mu_o IR^2}{2R^3}$$

and the magnetic field B at the center of the circular current loop is

$$B = \frac{\mu_o I}{2R}$$

Notice that this is the same solution we obtained in section 26.6, equation 26.35. Notice that this solution, equation 26.41, is more general than the one we derived in section 26.6.

26.8 Ampere's Circuital Law

Although the Biot-Savart law can be used to determine the magnetic field for different current distributions many of the derivations require vector integrations. Another simpler technique for the computation of magnetic fields, when the symmetry is appropriate, is Ampere's Circuital Law. *Ampere's Law states: along any arbitrary path encircling a total current* I_{total}, *the integral of the scalar product of the* *magnetic field* **B** *with the element of length d**l** of the path, is equal to the permeability* μ_o *times the total current* I_{total} *enclosed by the path.*[1] That is,

$$\oint \boldsymbol{B} \bullet d\boldsymbol{l} = \mu_o I_{\text{total}} \tag{26.42}$$

It should be pointed out that Ampere's law is a fundamental law based on experiments and cannot be derived. Ampere's law is especially helpful in problems with symmetry and will be used in the following sections.

26.9 The Magnetic Field Around a Long Straight Wire by Ampere's Law

To determine the magnetic field around a long straight wire by the Biot-Savart law is a little complicated, as seen in section 26.7. However, because of the symmetry of the magnetic field around a long straight wire, a simple solution to the magnetic field can be found using Ampere's law. It was pointed out in section 26.4 that the magnetic field around a long straight wire was found by experiment to be circular as shown in figure 26.14(a). In applying Ampere's law, an arbitrary path must be

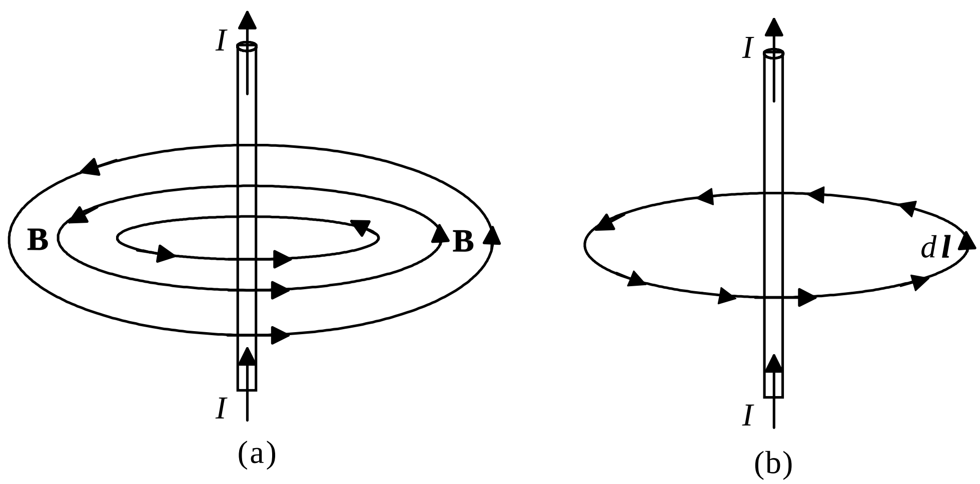


Figure 26.14 The magnetic field around a long straight wire.

drawn around the wire that contains the current I. The most symmetrical path that can be drawn about the wire is a concentric circle, as shown in figure 26.14(b). This circular path is divided into elements $d\boldsymbol{l}$, and then $\mathbf{B} \bullet d\boldsymbol{l}$ is computed for each element $d\boldsymbol{l}$. Because the magnetic field is circular, **B** and $d\boldsymbol{l}$ are parallel at every point along the circular path. Ampere's law becomes

[1] In many problems the subscript total will be left out of the current term I in the statement of Ampere's law, but it must be understood that the term I in Ampere's law is always the *total current* enclosed in the path of integration.

$$\oint \mathbf{B} \bullet d\mathbf{l} = \mu_o I$$
$$\oint \mathbf{B} \bullet d\mathbf{l} = \oint Bdl \cos 0^0 = \oint Bdl = \mu_o I \qquad (26.43)$$

But from symmetry, the value of B is the same anywhere along the circular path and can be taken outside of the integral to yield

$$B \oint dl = \mu_o I \qquad (26.44)$$

But the sum of all the dl's, that is, $\int dl$, is the circumference of the circular path. Hence,

$$\int dl = 2\pi r \qquad (26.45)$$

Therefore,

$$B(2\pi r) = \mu_o I \qquad (26.46)$$

Thus, *the magnitude of the magnetic field around a long straight wire is*

$$B = \frac{\mu_o I}{2\pi r} \qquad (26.47)$$

Example 26.9

The magnetic field of a long, straight wire. A long straight wire is carrying a current of 15.0 A. Find the magnetic field 30.0 cm from the wire.

Solution

The magnetic field around the wire is found from equation 26.47 to be

$$B = \frac{\mu_o I}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{T m/A})(15.0 \text{ A})}{2\pi(0.300 \text{ m})}$$
$$B = 1.00 \times 10^{-5} \text{ T}$$

To go to this Interactive Example click on this sentence.

26.10 Force between Parallel, Current-Carrying Conductors -- The Definition of the Ampere

Two conductors, carrying currents I_1 and I_2, respectively, are separated by a distance r, as shown in figure 26.15. The current I_2 in wire 2 produces a magnetic

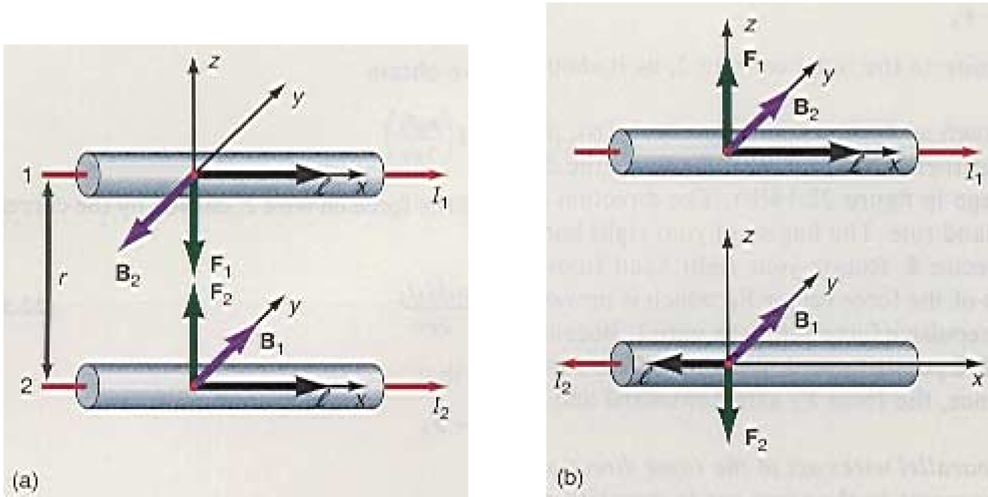


Figure 26.15 Force on parallel conductors carrying currents.

field $\mathbf{B}_2$ at the location of wire 1. Its magnitude, given by equation 26.47, is

$$B_2 = \frac{\mu_0 I_2}{2\pi r} \tag{26.48}$$

The direction of this magnetic field is found using the right-hand wire rule. If you grasp the wire with your thumb in the direction of the current, then your fingers curl in the direction of the magnetic field. In this case, B_2 at the location of wire 1 has a direction coming out of the paper, as shown in figure 26.15(a). Since wire 1 is a current-carrying wire in the external magnetic field $\mathbf{B}_2$, it experiences a force given by equation 26.17 as

$$F_1 = I_1 l B_2 \sin \theta$$

Using the same technique as in section 26.2, the direction of the force is again found by the right-hand rule. The fingers of your right hand are pointed in the direction of the length vector $\boldsymbol{l}$, which is in the direction of the current, with your palm facing the magnetic field vector $\mathbf{B}_2$, which points out of the page in figure 26.15. Rotate your right hand from $\boldsymbol{l}$ to $\mathbf{B}_2$ and your thumb points in the direction of the force vector $\mathbf{F}_1$, which is downward in figure 26.15(a). Hence wire 1 experiences a force of attraction toward wire 2. The angle θ between $\boldsymbol{l}$ and $\mathbf{B}_2$ is 90^0 and since $\sin 90^0 = 1$, the magnitude of $\mathbf{F}_1$ is

$$F_1 = I_1 l B_2$$

Substituting for the magnetic field B_2 from equation 26.48, gives

$$F_1 = I_1 l \left(\frac{\mu_o I_2}{2\pi r}\right)$$

Rearranging terms, the force on wire 1 caused by wire 2 is

$$F_1 = \frac{\mu_0 l I_1 I_2}{2\pi r} \tag{26.49}$$

Just as there is a force on wire 1, there is also a force on wire 2, because the current flowing in wire 1 produces a magnetic field $\mathbf{B}_1$. Wire 2 finds itself in this external magnetic field and hence experiences the force

$$F_2 = I_2 l B_1 \sin\theta$$

The direction of the magnetic field $\mathbf{B}_1$ produced by wire 1 is found from the right-hand wire rule, and is shown going into the paper at the location of wire 2 in figure 26.15(a). The direction of the force $\mathbf{F}_2$ is found using the right-hand rule. The fingers of your right hand are pointed in the direction of the length vector $\boldsymbol{l}$, which is in the direction of the current. Your palm should be facing the magnetic field vector $\mathbf{B}_1$, which points into the page in figure 26.15. Rotate your right hand from $\boldsymbol{l}$ to $\mathbf{B}_1$ and your thumb points in the direction of the force vector $\mathbf{F}_2$, which is upward in figure 26.15(a). Hence the force $\mathbf{F}_2$ acting on wire 2 is one of attraction toward wire 1. Since $\boldsymbol{l}$ and $\mathbf{B}_1$ are perpendicular, the magnitude of the force $\mathbf{F}_2$ is

$$F_2 = I_2 l B_1 \tag{26.50}$$

The magnetic field B_1, found from equation 26.48, is

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \tag{26.51}$$

Substituting equation 26.51 into 26.50, we obtain

$$F_2 = I_2 l \frac{\mu_0 I_1}{2\pi r}$$

After rearranging terms, the magnitude of the force on wire 2, caused by the current flowing in wire 1, is

$$F_2 = \frac{\mu_0 l I_1 I_2}{2\pi r} \tag{26.52}$$

Observe from equations 26.49 and 26.52 that

$$F_1 = F_2$$

And, from figure 26.15,

$$\mathbf{F}_1 = -\mathbf{F}_2$$

Thus the force on wire 1 is equal and opposite to the force on wire 2, as it should be according to Newton's third law.

If one of the currents is reversed, such as I_2 shown in figure 26.15(b), the magnitudes of the forces do not change but their directions do. Changing the direction of I_2, causes $\mathbf{B}_2$ to point into the page in figure 26.15(b). The direction of the force $\mathbf{F}_1$ is again found using the right-hand rule. The fingers of your right hand are pointed in the direction of the length vector $\boldsymbol{l}$. Rotate your right hand from $\boldsymbol{l}$ to $\mathbf{B}_2$ and your thumb points in the direction of the force vector $\mathbf{F}_1$, which is upward in figure 26.15(b), indicating that there is a repulsive force acting on wire 1. Because of the new direction of the current in wire 2, $\boldsymbol{l}$ points toward the left, and the force now acts downward in figure 26.15(b). Hence, the force $\mathbf{F}_2$ acts downward and is now a repulsive force.

In summary, *if the currents in the parallel wires act in the same direction, the force on the wires is attractive; if the currents in the wires act in opposite directions, the force on the wires is repulsive.*

In chapter 1, we noted that electric charge is a fundamental characteristic of matter and should be listed as the fourth fundamental quantity along with length, mass, and time. However, because electric charge is relatively difficult to measure, electric current, which is relatively easy to measure, is used as the fourth fundamental quantity. (Obviously charge is more fundamental than current in the description of nature because charges can exist at rest, which is equivalent to no current at all. That is, a charge can exist even when a current does not.) *The fundamental unit of electricity is defined as the **ampere,** where the ampere is that constant current that, if maintained in two straight parallel conductors of infinite length, of negligible circular cross section, and placed 1 m apart in a vacuum, would produce between these conductors a force equal to 2* $\times 10^{-7}$ N/m.

This force per unit length can be obtained from equation 26.52 with $I_1 = I_2 = I$, and $r = 1$ m, as

$$\frac{F}{l} = \frac{\mu_0 I^2}{2\pi(1\text{ m})}$$

For $I = 1$ A, the force is

$$\frac{F}{l} = \frac{(4\pi \times 10^{-7}\text{ T m/A})(1\text{ A})^2}{2\pi(1\text{ m})} = 2 \times 10^{-7}\text{ T A}$$

$$= (2 \times 10^{-7}\text{ T A})\left(\frac{\frac{\text{N}}{\text{A m}}}{\text{T}}\right)$$

$$= 2 \times 10^{-7}\ \frac{\text{N}}{\text{m}}$$

That is, the amount of current that causes this force is defined as the ampere.

Example 26.10

Force on parallel conducting wires. Two straight parallel conductors 30.0 cm long are separated by a distance of 10.0 cm. If one carries a current of 5.00 A to the right while the second carries a current of 7.00 A to the left, find the force on the first wire.

Solution

The magnitude of the force on the first wire, found from equation 26.49, is

$$F_1 = \frac{\mu_0 l I_1 I_2}{2\pi r}$$

$$= \frac{(4\pi \times 10^{-7}\ \text{T m/A})(0.300\ \text{m})(5.00\ \text{A})(7.00\ \text{A})}{2\pi(0.100\ \text{m})}$$

$$= 2.10 \times 10^{-5}\ \text{N}$$

and is repulsive.

To go to this Interactive Example click on this sentence.

26.11 The Magnetic Field Inside a Solenoid

A solenoid is a long coil of wire with many turns and is shown schematically in figure 26.16. Note that the magnetic field of a solenoid looks like the magnetic field

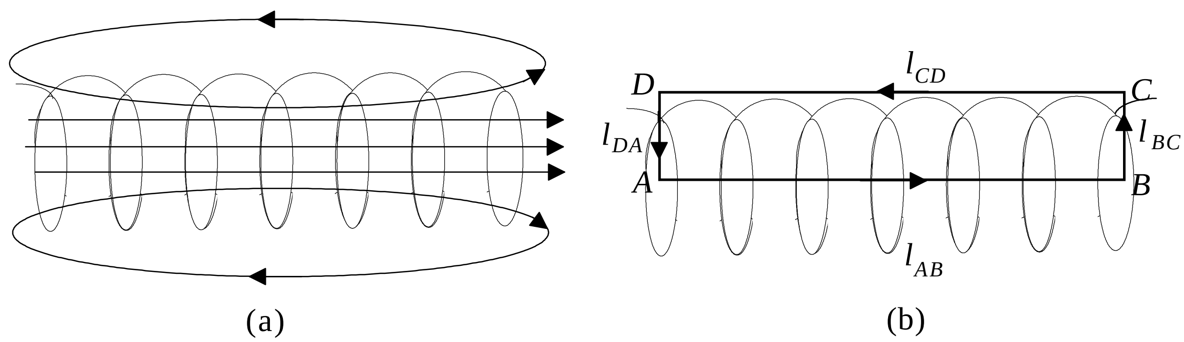


Figure 26.16 The magnetic field of a solenoid

of a bar magnet. The magnetic field is uniform and intense within the coils but is so small outside of the coil, that it is taken to be zero there. The magnetic field inside the solenoid lies along the axis of the coil, as seen in figure 26.16(a). The value of **B** inside the solenoid is found from Ampere's law, by adding up the values of $\mathbf{B} \bullet d\mathbf{l}$ along the rectangular path *ABCD* in figure 26.16(b). That is

$$\oint \boldsymbol{B} \bullet d\boldsymbol{l} = \mu_o I_{\text{total}}$$

$$\int_{AB} \boldsymbol{B} \bullet d\boldsymbol{l} + \int_{BC} \boldsymbol{B} \bullet d\boldsymbol{l} + \int_{CD} \boldsymbol{B} \bullet d\boldsymbol{l} + \int_{DA} \boldsymbol{B} \bullet d\boldsymbol{l} = \mu_o I_{\text{total}} \tag{26.53}$$

But since **B** = 0 outside the solenoid

$$\int_{CD} \boldsymbol{B} \bullet d\boldsymbol{l} = 0 \tag{26.54}$$

Because **B** is perpendicular to the paths $\boldsymbol{l}_{BC}$ and $\boldsymbol{l}_{DA}$, their scalar products must be zero, i.e.,

$$\int_{BC} \boldsymbol{B} \bullet d\boldsymbol{l} = \int_{BC} Bdl \cos 90^0 = 0 \tag{26.55}$$

and

$$\int_{DA} \boldsymbol{B} \bullet d\boldsymbol{l} = \int_{DA} Bdl \cos 90^0 = 0 \tag{26.56}$$

Now **B** is parallel to $\boldsymbol{l}_{AB}$, therefore their scalar product becomes

$$\int_{AB} \boldsymbol{B} \bullet d\boldsymbol{l} = \int_{AB} Bdl \cos 0^0 = \int_{AB} Bdl = Bl_{AB} \tag{26.57}$$

Replacing equations 26.54 through 26.57 into equation 26.53 gives

$$Bl_{AB} = \mu_o I_{\text{total}} \tag{26.58}$$

The term, I_{total}, represents the total amount of current contained within the path *ABCD*. Each turn of wire carries a current I, but there are N turns of wire in the solenoid. Therefore, the total current contained within the path is

$$I_{\text{total}} = NI \tag{26.59}$$

A more convenient unit for the number of turns of wire in the coil is the number of turns per unit length, n. Since the coil has a length l_{AB}, the total number of turns N can be expressed as

$$N = nl_{AB} \tag{26.60}$$

Replacing equations 26.60 and 26.59 into equation 26.58 yields

$$Bl_{AB} = \mu_o I_{\text{total}} = \mu_o NI = \mu_o nl_{AB} I \tag{26.61}$$

Simplifying, the magnetic field within the solenoid becomes

$$B = \mu_o nI \tag{26.62}$$

Notice that the magnetic field within the solenoid can be increased by increasing the current I flowing in the wires, and/or by increasing the number of turns of wire per unit length, n.

Example 26.11

Magnetic field inside a solenoid. A solenoid 15.0 cm long is composed of 300 turns of wire. If there is a current of 5.00 A in the wire, what is the magnetic field inside the solenoid?

Solution

The number of turns of wire per unit length is

$$n = \frac{N}{l} = \frac{300 \text{ turns}}{0.150 \text{ m}} = 2000 \text{ turns/m}$$

The magnetic field inside the solenoid is found from equation 26.62 as

$$B = \mu_o nI = (4\pi \times 10^{-7} \frac{\text{T m}}{\text{A}})(\frac{2000 \text{ turns}}{\text{m}})(5.00 \text{ A})$$
$$B = 1.26 \times 10^{-2} \text{ T}$$

To go to this Interactive Example click on this sentence.

26.12 Magnetic Field Inside a Toroid

Let us now find the magnetic field inside a toroid. A toroid is essentially a solenoid of finite length bent into the shape of a doughnut as shown in figure 26.17. To

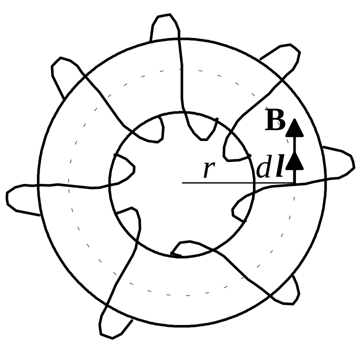


Figure 26.17 The magnetic field inside a toroid.

compute the magnetic field within the toroid we use Ampere's law, equation 26.42.

$$\oint \boldsymbol{B} \bullet d\boldsymbol{l} = \mu_o I_{\text{total}} \tag{26.42}$$

Since the toroid is a solenoid bent into a circular shape, and the magnetic field of a solenoid is parallel to the walls of the solenoid, the magnetic field **B** inside the toroid will take the same shape as the walls of the toroid. Hence the magnetic field inside the toroid will be circular. The path that we will use for the line integral in Ampere's law will be a circle of radius *r* as shown in the diagram. In this way the angle θ between the magnetic field vector **B** and the element $d\boldsymbol{l}$ is 0^0. Thus Ampere's law becomes

$$\oint \boldsymbol{B} \bullet d\boldsymbol{l} = \int B dl \cos 0^0 = \int B dl = \mu_o I_{\text{total}} \tag{26.63}$$

But from the symmetry of the problem, the magnitude of the magnetic field *B* is a constant at a particular radius r from the center of the toroid. (That is, there is no reason to assume that the magnitude of the magnetic field *B* a distance *r* to the right of the center of the toroid should be any different than the magnitude of the magnetic field *B* a distance *r* to the left of the center of the toroid.) Hence we can take B outside the integral sign in equation 26.63 to obtain

$$\int B dl = B \int dl = \mu_o I_{\text{total}} \tag{26.64}$$

We now see that the integration is only over the element of path *dl* and the sum of all these *dl*'s is just the circumference of the circle of radius *r*. Therefore Ampere's law becomes

$$B \int dl = B(2\pi r) = \mu_o I_{\text{total}} \tag{26.65}$$

Now the current term I_{total} represents the total amount of current contained within the path of integration. Each turn of wire carries a current *I*, but there are *N* turns of wire in the toroid. Therefore, the total current contained within the path of integration is

$$I_{\text{total}} = NI \tag{26.66}$$

Replacing equation 26.66 into equation 26.65 yields

$$B(2\pi r) = \mu_o I_{\text{total}} = \mu_o NI \tag{26.67}$$

Upon solving for *B* we get

$$B = \frac{\mu_o NI}{2\pi r} \tag{26.68}$$

Equation 26.68 gives the value of the magnetic field B inside the toroid. Notice that the value of B is not constant over the cross section of the toroid, but rather is a function of r, the distance from the center of the toroid (specifically B varies as 1/r). For the smaller values of *r*, near the inside of the toroid, *B* will be large and for the larger values of *r*, near the outside of the toroid, *B* will be smaller. *Hence, the magnetic field B inside a toroid is not constant and uniform as it was in the solenoid.*

Example 26.12

The magnetic field inside a toroid. A toroid has an inner radius of 10.0 cm and an outer radius of 20.0 cm and carries 500 turns of wire. If the current in the toroid is 5.00 A, find the minimum and maximum values of the magnetic field inside the toroid.

Solution

The minimum value of the magnetic field B within the toroid occurs for the maximum value of r and is found from equation 26.68 as

$$B = \frac{\mu_o NI}{2\pi r}$$
$$B = \frac{(4\pi \times 10^{-7}\,\text{T m/A})(500)(5.00\text{ A})}{2\pi(0.200\text{ m})}$$
$$B = 0.0025\text{ T}$$

The maximum value of the magnetic field B within the toroid occurs for the minimum value of r and is found from equation 26.68 as

$$B = \frac{\mu_o NI}{2\pi r}$$
$$B = \frac{(4\pi \times 10^{-7}\,\text{T m/A})(500)(5.00\text{ A})}{2\pi(0.100\text{ m})}$$
$$B = 0.0050\text{ T}$$

To go to this Interactive Example click on this sentence.

26.13 Torque on a Current Loop in an External Magnetic Field - The Magnetic Dipole Moment

Let us place a rectangular coil of wire in a uniform magnetic field B as shown in figure 26.18(a). Notice that the magnetic field emanates from the north pole of the magnet and enters the south pole of the magnet. A current I is set up in the coil in the direction indicated, by an external battery. Any segment of the coil now represents a current carrying wire in an external magnetic field and will thus experience a force on it given by equation 26.16.

$$\mathbf{F} = I\,\boldsymbol{l} \times \mathbf{B} \qquad (26.16)$$

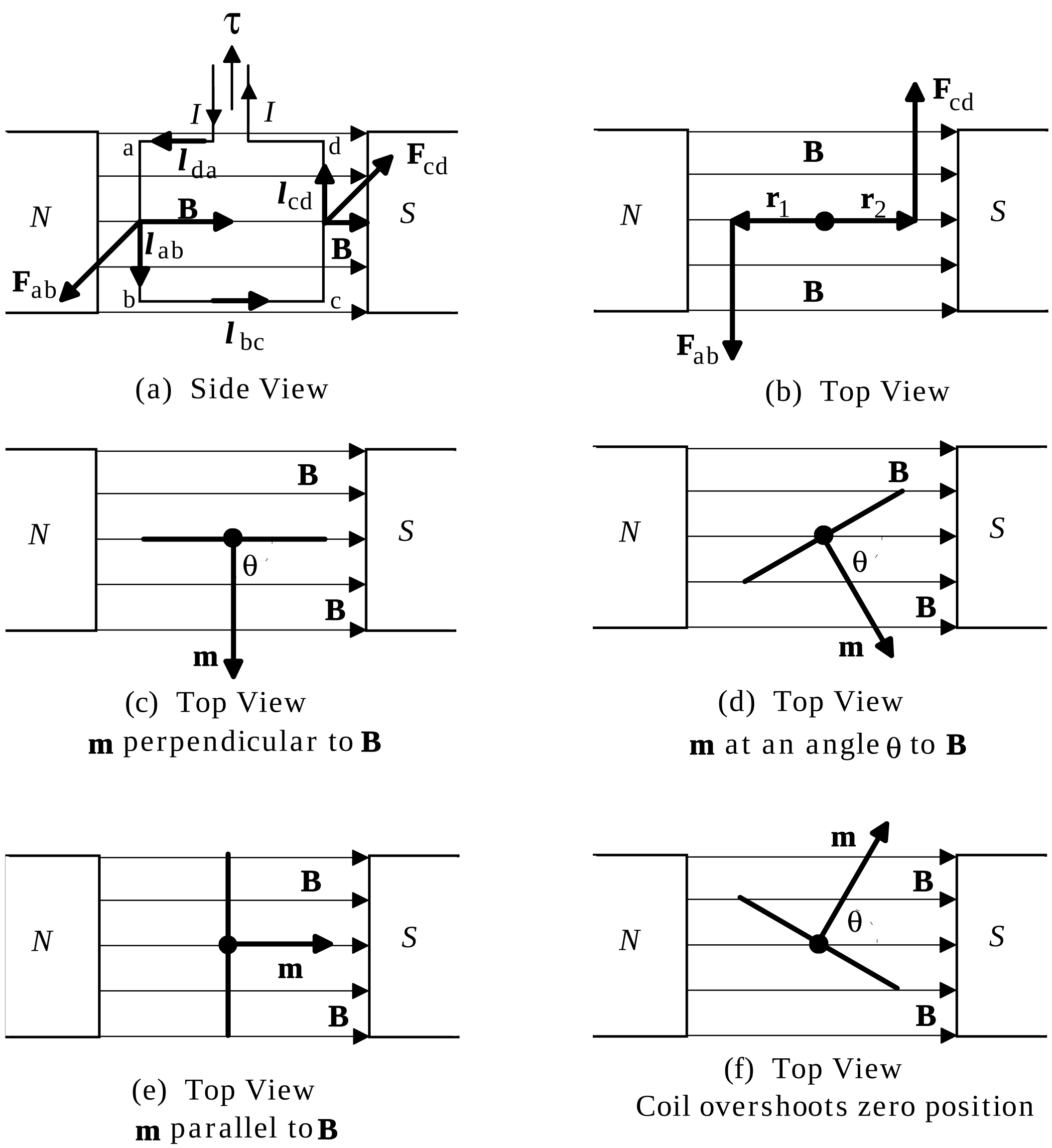


Figure 26.18 A coil in a magnetic field.

Let us divide the coil into sections of length l_{ab}, l_{bc}, l_{cd}, l_{da} and compute the force on each segment.

Segment ab: The force $\mathbf{F}_{ab}$ acting on segment l_{ab} is given by

$$\mathbf{F}_{ab} = I\,\boldsymbol{l}_{ab} \times \mathbf{B} \tag{26.69}$$

Since $\boldsymbol{l}_{ab}$ is in the direction of the current in segment ab, it points downward in figure 26.18(a). The cross product of $\boldsymbol{l}_{ab} \times \mathbf{B}$ points outward as shown in the side view of the coil in figure 26.18(a), and from a top view in figure 26.18(b). The magnitude of the force is

$$F_{ab} = Il_{ab}B \sin 90^0 = Il_{ab}B \tag{26.70}$$

Segment bc: The force $\mathbf{F}_{bc}$ acting on segment $\boldsymbol{l}_{bc}$ is given by

$$\mathbf{F}_{bc} = I\boldsymbol{l}_{bc} \times \mathbf{B} \tag{26.71}$$

Since $\boldsymbol{l}_{bc}$ is in the direction of the current flow, it points to the right in figure 26.18(a), parallel to the magnetic field **B**. As you recall from chapter 1, the cross product of parallel vectors is zero. That is

$$F_{bc} = Il_{bc}B \sin 0^0 = 0 \tag{26.72}$$

Thus, there is no force acting on the bottom of the wire coil in figure 26.18(a).
Segment cd: The force $\mathbf{F}_{cd}$ acting on segment $\boldsymbol{l}_{cd}$ is given by

$$\mathbf{F}_{cd} = I\boldsymbol{l}_{cd} \times \mathbf{B} \tag{26.73}$$

Since $\boldsymbol{l}_{cd}$ points upward and **B** points to the right in figure 26.18(a), the cross product $\boldsymbol{l}_{cd} \times \mathbf{B}$ points into the page in figure 26.18(a), and can be seen from the top view in figure 26.18(b). The magnitude of F_{cd} is

$$F_{cd} = Il_{cd}B \sin 90^0 = Il_{cd}B \tag{26.74}$$

Because the magnitudes of the lengths l_{ab} and l_{cd} are equal, the forces are also equal. That is,

$$F_{ab} = F_{cd}$$

Segment da: The force $\mathbf{F}_{da}$ acting on the upper segment $\boldsymbol{l}_{da}$ is found from

$$\mathbf{F}_{da} = I\boldsymbol{l}_{da} \times \mathbf{B} \tag{26.75}$$

But $\boldsymbol{l}_{da}$ points to the left and makes an angle of 180^0 with the magnetic field **B**. Therefore,

$$F_{da} = Il_{da}B \sin 180^0 = 0 \tag{26.76}$$

because the $\sin 180^0$ is equal to zero. Hence, there is no force acting on the top wire.

The net result of the current flowing in a coil in an external magnetic field, is to produce two equal and opposite forces acting on the coil. But since the forces do not lie along the same line of action, the forces will cause a torque to act on the coil as is readily observable in figure 26.18(b). The torque acting on the coil is given by equation 9.72 as

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} \tag{9.72}$$

Since both forces can produce a torque, the total torque on the coil is the sum of the two torques. That is,

$$\boldsymbol{\tau} = \mathbf{r}_1 \times \mathbf{F}_{ab} + \mathbf{r}_2 \times \mathbf{F}_{cd} \qquad (26.77)$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are position vectors from the axis of the coil to the point of application of the force, as can be seen in figure 26.18(b). Therefore,

$$\mathbf{r}_2 = -\mathbf{r}_1 \qquad (26.78)$$

and

$$\mathbf{F}_{cd} = -\mathbf{F}_{ab} \qquad (26.79)$$

as shown before. Therefore,

$$\boldsymbol{\tau} = \mathbf{r}_1 \times \mathbf{F}_{ab} + (-\mathbf{r}_1) \times (-\mathbf{F}_{ab}) \qquad (26.80)$$

The total torque on the coil is therefore the clockwise torque

$$\boldsymbol{\tau} = 2\mathbf{r}_1 \times \mathbf{F}_{ab} \qquad (26.81)$$

and lies along the axis of the coil, pointing upward in figure 26.18(a). As can be seen from the diagram,

$$r_1 = \frac{l_{bc}}{2} \qquad (26.82)$$

while F_{ab} is given by equation 26.70. Therefore the magnitude of the torque on the coil is given by

$$\tau = 2r_1 F_{ab} \sin\theta = 2\frac{l_{bc}}{2} I l_{ab} B \sin\theta \qquad (26.83)$$

Note that the angle θ is the angle between the radius vector **r** and the force vector **F** and the product of l_{bc} and l_{ab} is the area of the loop, i.e.,

$$l_{bc} l_{ab} = A \qquad (26.84)$$

Equation 26.83 simplifies to

$$\tau = IAB\sin\theta \qquad (26.85)$$

Although equation 26.85 has been derived for a rectangular current loop, one can prove, that it is a perfectly general result for any planar loop having an area A, regardless of the shape of the loop. *The torque on the coil in a magnetic field depends on the current I in the coil, the area A of the coil, the intensity of the magnetic field B, and the angle* θ *between* **r** *and* $\mathbf{F}_{ab}$. Since it was shown in section 26.6 that there is a magnetic field at the center of a current loop, and this magnetic field looks like the magnetic field of a bar magnet, *a magnetic dipole moment* $\boldsymbol{m}$ *of a current loop is defined as*

$$\boldsymbol{m} = IA\boldsymbol{n} \qquad (26.86)$$

The magnetic dipole moment is shown in figure 26.19. **n** is a unit vector that determines the direction of **m**, and is itself determined by the direction of the current flow. If the right hand curls around the loop in the direction of the current

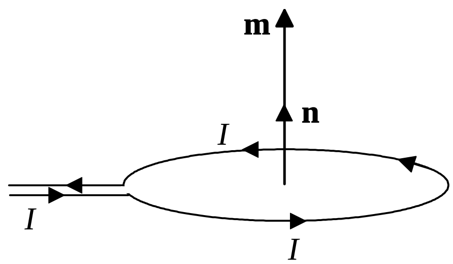


Figure 26.19 The magnetic dipole moment of a current loop.

flow, the thumb points in the direction of the unit vector **n**, and hence in the direction of the magnetic dipole moment **m**. Reversing the direction of the current flow will reverse the direction of the magnetic dipole moment. If the coil consists of N loops of wire, the magnetic dipole moment will be given by

$$\mathbf{m} = NIA\,\mathbf{n} \tag{26.87}$$

Figure 26.18(c) is a top view of the coil, showing the magnetic dipole moment **m** perpendicular to the coil. The torque acting on the coil, equation 26.85, can now be written in terms of the magnetic dipole moment of the coil as

$$\tau = mB\sin\theta \tag{26.88}$$

or in the vector notation

$$\boldsymbol{\tau} = \boldsymbol{m} \times \boldsymbol{B} \tag{26.89}$$

Note that the angle θ in equation 26.89, the angle between the magnetic dipole moment vector **m** and the magnetic field vector **B** is the same angle θ that was between the radius vector **r** and the force vector **F**. *Equation 26.89 shows that the torque acting on a coil in a magnetic field is equal to the cross product of the magnetic dipole moment **m** of the coil and the magnetic field* **B**. When **m** is perpendicular to the magnetic field **B**, θ is equal to 90^0, and the sine of $90^0 = 1$. Therefore the torque acting on the coil will be at its maximum value, and will act to rotate the coil counterclockwise in figure 26.18(c). As the coil rotates, the angle θ will decrease, figure 26.18(d), until the magnetic dipole moment becomes parallel to the magnetic field, and the angle θ will be equal to zero, as shown in figure 26.18(e). At this point the torque acting on the coil becomes zero, because the sinθ term in equation 26.88, will be zero. Because of the inertia of the coil, however, the coil does not quite stop at this position but over shoots this position as shown in figure 26.18(f). But now the torque, given by **m** × **B**, is reversed, and the coil will rotate clockwise, until **m** is again parallel to **B** and again the torque will be zero. The coil may oscillate one or two times but because of friction it eventually will come to a stop with its magnetic dipole moment parallel to the magnetic field **B**. *In summary, the magnetic field **B** will cause a torque* **τ** *to act on the coil until the magnetic dipole moment **m** of the coil is aligned with the magnetic field **B**.*

This result should not come as too much of a surprise, for this is exactly what happens with a compass needle. The compass needle is a tiny bar magnet with a magnetic dipole moment. The earth's magnetic field acts on this dipole to align it with the earth's magnetic field. On the earth's surface, the earth's magnetic field points toward the north and the compass needle will also point toward the north.

Example 26.13

The magnetic dipole moment of a coil. A circular coil, consisting of 10 turns of wire, 10.0 cm in diameter carries a current of 2.00 A. Find the magnetic dipole moment of the coil.

Solution

The area of the coil is

$$A = \frac{\pi d^2}{4} = \frac{\pi(0.100\text{ m})^2}{4} = 7.85 \times 10^{-3}\text{ m}^2$$

The magnitude of the magnetic dipole moment of the coil is found from equation 26.87 as

$$m = NIA$$
$$m = (10)(2.00\text{ A})(7.95 \times 10^{-3}\text{ m}^2)$$
$$m = 0.157\text{ A m}^2$$

Notice that the unit for magnetic dipole moment is an ampere meter squared.

To go to this Interactive Example click on this sentence.

Example 26.14

The torque on a magnetic dipole in an external magnetic field. The above coil is placed in a uniform magnetic field of 0.500 T. Find the maximum torque on the coil.

Solution

The torque acting on the coil is found from equation 26.88 as

$$\tau = mB \sin\theta$$

and the maximum torque occurs for $\theta = 90^0$. Therefore, the maximum torque acting on the coil is

$$\tau_{max} = mB = (0.157 \text{ A m}^2)(0.500 \text{ T})$$
$$\tau_{max} = (7.85 \times 10^{-2} \text{ A m}^2 \text{ T})\left(\frac{\text{N/(A m)}}{\text{T}}\right)$$
$$\tau_{max} = 7.85 \times 10^{-2} \text{ m N}$$

To go to this Interactive Example click on this sentence.

26.14 Applications of the Torque on a Current Loop in an External Magnetic Field

The fact that a torque acts on a current loop in a magnetic field gives rise to a number of important applications. Among them are the DC motor, and the galvanometer.

The DC Motor

We saw in section 26.13 that when a current flows in a coil in an external magnetic field **B,** a torque arises, given by equation 26.89, that causes the magnetic dipole moment of the coil to be aligned with the magnetic field, as in figures 26.18(c), 26.18(d), and 26.18(e). Because of the inertia of the coil, the coil passes the aligned position, as shown in figure 26.18(f). Then the torque is reversed and the coil rotates back to the aligned position. When the coil overshoots the aligned position, figure 26.18(f), if the direction of the current were to simultaneously change, the direction of the magnetic dipole moment of the coil would be reversed, as shown in figure 26.18(g). The torque τ would again point upward, indicating that the coil will continue to rotate in a counterclockwise direction away from the aligned position. If we continue to change the direction of the current, and hence the direction of the magnetic dipole moment, every time the magnetic dipole moment passes the aligned position, the coil will rotate indefinitely. To accomplish this change in current every 180^0 of rotation we use a split-ring commutator, as shown schematically in figure 26.20. In the position shown, current from the battery flows through the coil as indicated. When the first split ring rotates past the brush, the other side of the coil gets connected to the positive terminal of the battery and the current flows in the opposite direction. The split ring is thus responsible for changing the direction of the current, and hence the direction of the magnetic dipole moment. The coil continues to rotate as long as the current flows and this is the essence of a DC motor.

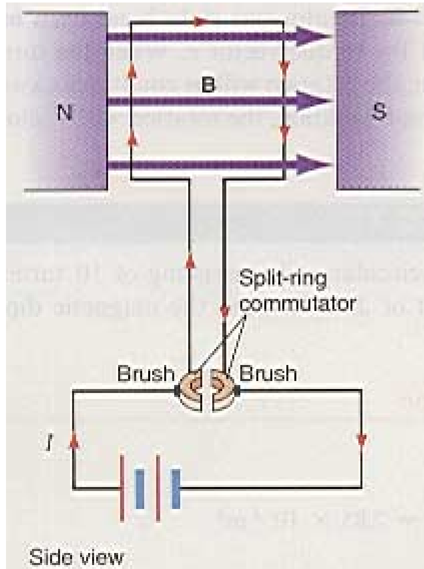


Figure 26.20 The DC motor.

The Galvanometer

A torque acting on a current loop in an external magnetic field is the basis for the d'Arsonval galvanometer named after the French physicist Jacques Arsène d'Arsonval (1851-1940). The galvanometer is used to measure current in a circuit. A simplified version of a galvanometer is shown schematically in figure 26.21. With a

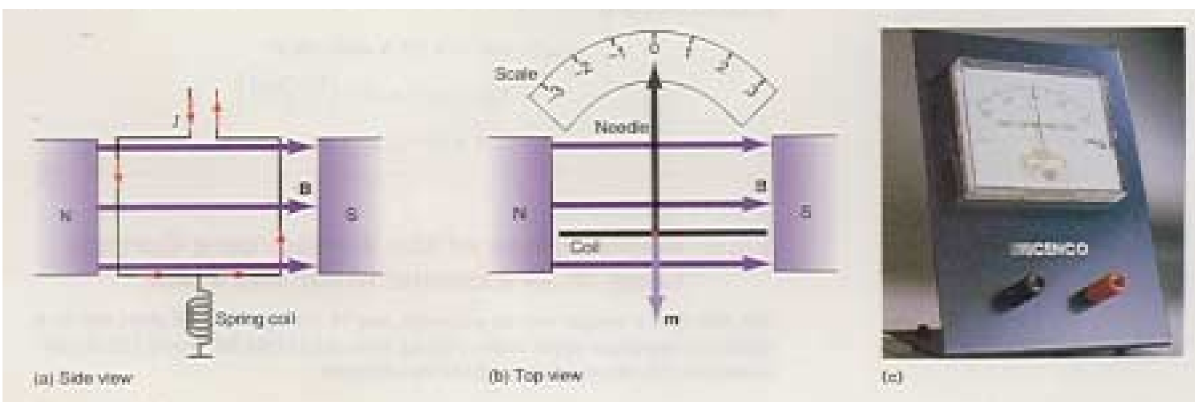


Figure 26.21 The galvanometer.

current I in the coil, a torque τ is produced in the coil, whose magnitude is given by

$$\tau = mB \sin \theta \qquad (26.88)$$

This torque causes the coil to rotate. A spring coil is attached to the bottom of the coil and causes an opposite reaction torque to be developed given by

$$\tau_{spring} = K\phi$$

where K is the torsional constant of the spring and ϕ is the rotational angle turned through by the coil. The coil rotates until there is an equilibrium between the torque on the coil and the restoring torque of the spring. A needle is attached to the top of the coil and moves over a scale as the coil turns. This is illustrated in figure 26.21(b). At equilibrium,

$$\tau_{spring} = \tau_{coil}$$

Therefore,

$$K\phi = mB \sin\theta$$

In the position shown in figure 26.21, $\theta = 90^0$, and $\sin 90^0 = 1$. Therefore,

$$K\phi = mB$$

But the magnitude of the magnetic dipole moment of the coil is found from equation 26.85 as

$$m = IA$$

Therefore, the angle that the coil turns through before coming to rest is

$$\phi = \frac{IAB}{K} = \left(\frac{AB}{K}\right) I \tag{26.90}$$

The angle ϕ thus depends on the area A of the coil, the magnetic field B, the torsional constant K of the spring, and the current flowing in the loop. Since A, B, and K are constants for a particular loop *the angle that the coil turns through, equation 26.90, is directly proportional to the current flowing in the coil.* This gives us a means for measuring the current in a circuit. Because of the uniform magnetic field B shown in figure 26.21, equation 26.90 is good for only very small deflections. In actual galvanometers, a cylindrical core is placed within the coil, and the pole faces of the magnet are usually curved so as to supply a radial magnetic field. Thus the coil always has **m** perpendicular to **B,** the swing of the coil is independent of $\sin\theta$, and the swing is proportional to the current I throughout the range of the meter.

The galvanometer, however, reads only relative currents (that is, where I_1 is twice as great as I_0, I_2 is five times as large as I_0, etc.). The galvanometer must be calibrated to read a current in amperes. Most galvanometers are rated in terms of their electrical resistance R_g and the amount of current I_g that is necessary for a full-scale deflection of the galvanometer. A typical student galvanometer is shown in figure 26.21(c).

26.15 Permanent Magnets and Atomic Magnets

The magnetic field of a bar magnet is shown in figure 26.22(a). The magnetic field emanates from the north pole of the magnet and enters at the south pole. The field

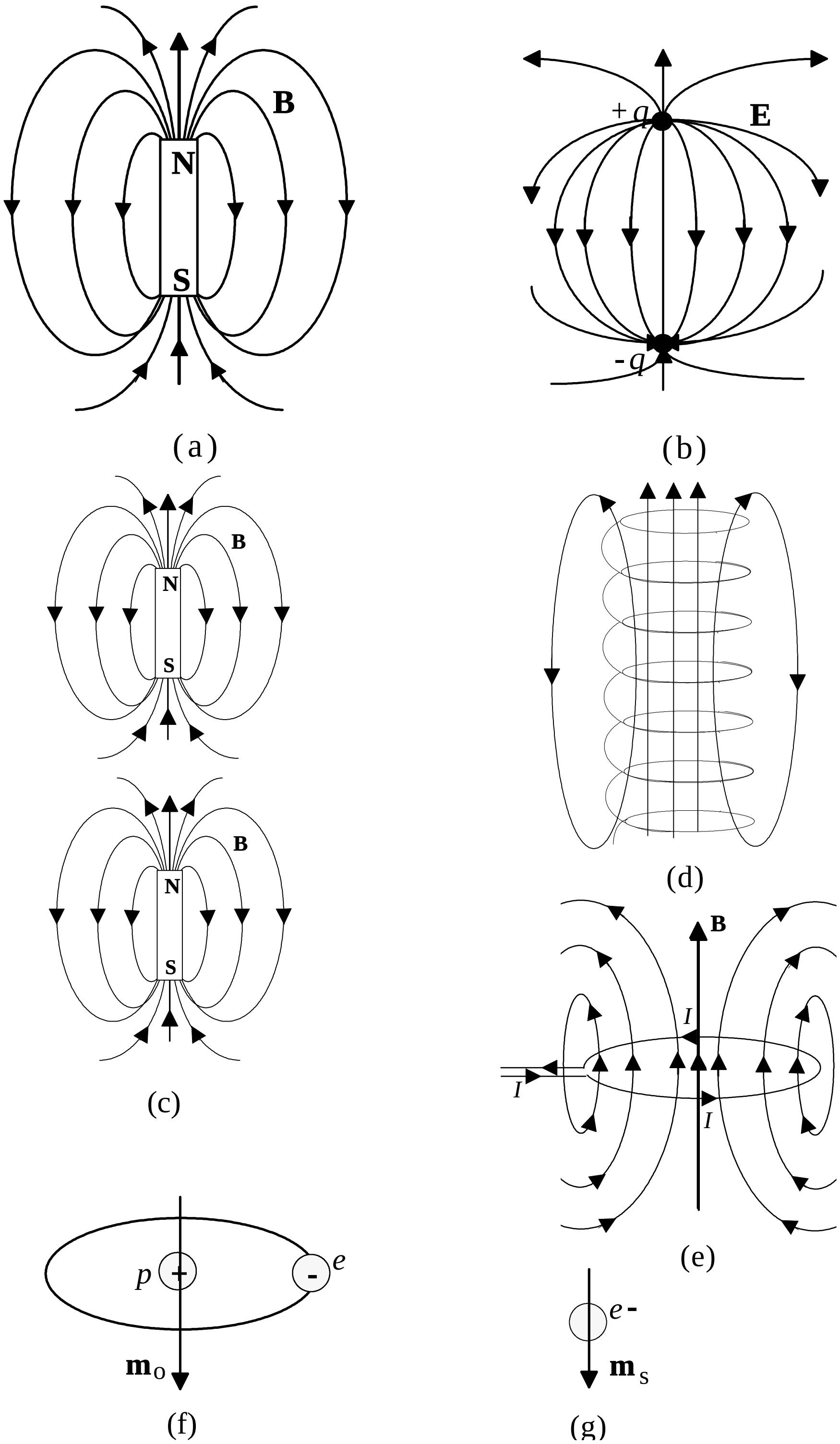


Figure 26.22 Some Magnetic Fields

is similar to the field of an electric dipole, shown in figure 26.22(b). The electric field emanates from the positive electric charge and terminates at the negative electric charge. Since both negative and positive electric charges can exist separately, it is reasonable to ask if magnetic poles can exist separately? The simplest test, would be to cut the bar magnet in figure 26.22(a) in half, expecting to obtain one isolated north pole and one isolated south pole. When the experiment is performed, however, two smaller bar magnets are found, each with a north pole and a south pole as seen in figure 26.22(c). *No matter how many times the bar magnet is subdivided, an isolated magnetic pole is never found. We always get a dipole.* Looking at the magnetic field of a solenoid, figure 26.22(d), it appears to be the same as the magnetic field of a bar magnet. So, maybe all magnetic fields are caused by electric currents. The magnetic field of a single current loop is shown in figure 26.22(e). Looking at a picture of the atom, we find the negative electron orbiting around the positive nucleus. The orbiting electron, figure 26.22(f), looks exactly like a current loop, and there is, therefore, a magnetic dipole moment of the atom, caused by the orbital electron. This orbital magnetic dipole moment is given by

$$m_o = IA \tag{26.91}$$

The atomic current is equal to the charge on the electron divided by the time T for the electron to make one complete orbit, i.e.,

$$I = \frac{(-e)}{T} = -ef \tag{26.92}$$

where f, the reciprocal of the period, is the frequency, or the number of times the electron orbits the nucleus per second. Assuming the orbit to be a circle, it has an area

$$A = \pi r^2 \tag{26.93}$$

Hence, the orbital magnetic moment of an atom is

$$m_o = IA = -ef\pi r^2 \tag{26.94}$$

Therefore the atom itself looks like a tiny bar magnet.

Besides the orbital magnetic moment of the atom, the electron, which can be viewed as a charged sphere spinning on its axis, also has a magnetic dipole moment associated with its spin. The spin magnetic dipole moment is represented by $\mathbf{m}_s$. In general the total magnetic dipole moment of an atom is equal to the vector sum of its orbital magnetic dipole moment $\mathbf{m}_o$ and its spin magnetic dipole moment $\mathbf{m}_s$. The electrons usually fill up the shells of an atom with one spin magnetic dipole moment pointing up, and the next down. Thus, when an atom has completely closed shells, it has no magnetic dipole moment. Therefore, most chemical elements do not display magnetic behavior. In the case of iron, cobalt, and nickel, the electrons do not pair off to permit their spin magnetic dipole moments to cancel. In iron, for example, five of its electrons have parallel spins, giving it a large resultant magnetic dipole moment. Therefore each atom of iron is a tiny atomic bar magnet. When these atomic magnets are aligned in a bar of iron, we have the common bar magnet. Thus, the bar magnet is made up of atomic currents. ***This is why we can never isolate a north pole or a south pole, because they do not exist.*** *What does exist is an orbiting, rotating electric charge, that creates a magnetic dipole moment.*

26.16 The Potential Energy of a Magnetic Dipole in an External Magnetic Field

In section 26.13 we saw that a coil of wire of cross-sectional area A, with N turns, carrying a current I, has a magnetic dipole moment **m** given by equation 26.87 as

$$\mathbf{m} = NIA\,\mathbf{n} \tag{26.87}$$

We also saw that when that coil of wire is placed in an external magnetic field **B** its magnetic dipole **m** experiences a torque given by

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B} \tag{26.89}$$

This torque acts to rotate the dipole until it is aligned with the external magnetic field. Because the natural position of **m** is parallel to the field, as shown in figure 26.18(e) and again here in figure 26.23(a), work must be done to rotate **m** in the

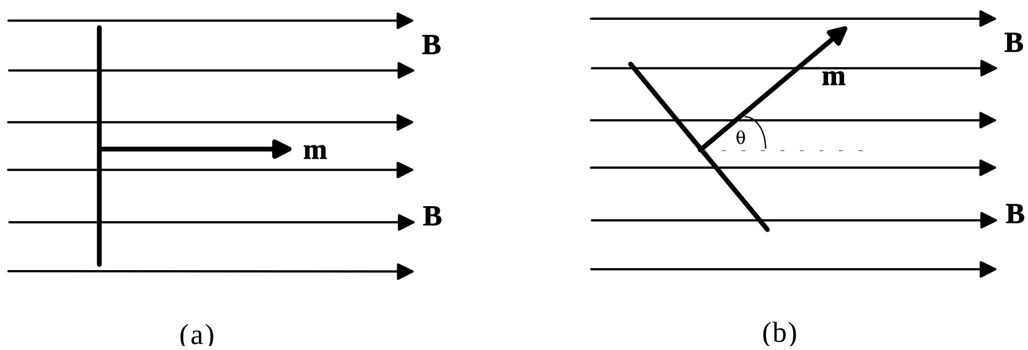


Figure 26.23 A magnetic dipole **m** in an external magnetic field **B**.

external magnetic field **B**. When work was done in lifting a rock in a gravitational field, the rock then possessed potential energy. In the same way, since work must be done by an external agent to change the orientation of the dipole, the work done in rotating the dipole in the magnetic field shows up as potential energy of the dipole, figure 26.23(b). That is, the dipole now possesses an additional potential energy associated with the work done in rotating **m**.

The potential energy of the dipole in an external magnetic field **B** is found by computing the work that must be done to rotate the dipole in the external magnetic field. That is,

$$\text{PE} = W = \int dW \tag{26.95}$$

Just as the element of work $dW = \mathbf{F} \bullet d\mathbf{s}$ for translational motion, the element of work for rotational motion is given by

$$dW = \boldsymbol{\tau} \bullet d\boldsymbol{\theta} \tag{26.96}$$

where $\boldsymbol{\tau}$ is the torque acting on the dipole to cause it to rotate and $d\boldsymbol{\theta}$ is the element of angle turned through. Both the torque vector $\boldsymbol{\tau}$ and the element of angle vector are perpendicular to the plane of the paper, and hence the angle between the two vectors are zero and their dot product is simply $\tau\, d\theta$. The increased potential energy becomes

$$\text{PE} = \int dW = \int \tau\, d\theta \tag{26.97}$$

The magnitude of the torque is found from equation 26.88 as

$$\tau = mB \sin\theta \tag{26.88}$$

Replacing equation 26.88 into equation 26.97 gives

$$\text{PE} = \int_{\theta_0}^{\theta} \tau d\theta = \int_{90^0}^{\theta} mB \sin\theta d\theta \tag{26.98}$$

We have made the lower limit of integration θ_o to be 90^0, and θ the upper limit. Equation 26.98 becomes

$$\text{PE} = -mB \cos\theta \,|_{90^0}^{\theta}$$

$$\text{PE} = -mB \cos\theta \tag{26.99}$$

Noticing the form of this equation, we can write it more generally as

$$\text{PE} = -\boldsymbol{m} \bullet \boldsymbol{B} \tag{26.100}$$

*Equation 26.100 gives the potential energy of a magnetic dipole **m** in an external magnetic field **B**.*

Example 26.15

The potential energy of a magnetic dipole. Find the potential energy of a magnetic dipole in an external magnetic field when (a) it is antiparallel to **B** (i.e., $\theta = 180^0$),

(b) it is perpendicular to **B** (i.e., $\theta = 90^0$), and (c) it is aligned with **B** (i.e., $\theta = 0$).

Solution

The potential energy of the dipole, found from equation 26.99, is

a.

$$PE = -\, mB \cos 180^0$$
$$PE = +\, mB$$

b.

$$PE = -\, mB \cos 90^0$$
$$PE = 0$$

c.

$$PE = -\, mB \cos 0^0$$
$$PE = -\, mB$$

Thus, the magnetic dipole has its highest potential energy when it is antiparallel (180^0), decreases to zero when it is perpendicular (90^0), and decreases to its lowest potential energy, a negative value, when it is aligned with the magnetic field, $\theta = 0^0$. This is shown in figure 26.24. So, just as the rock falls from a position of high potential energy to the ground where it has its lowest potential energy, the dipole, if given a slight push to get it started, rotates from its highest potential energy (antiparallel) to its lowest potential energy (parallel).

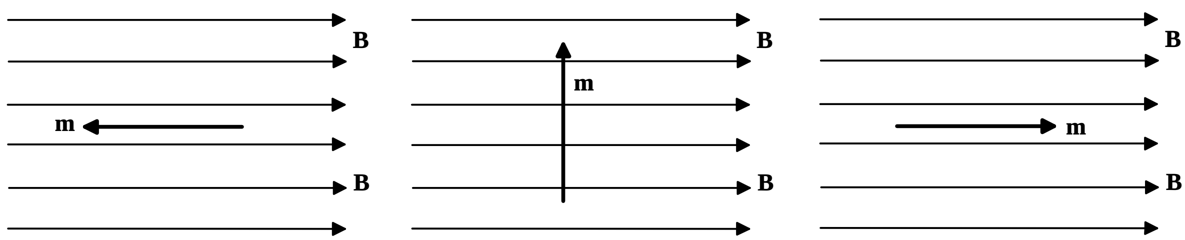


Figure 26.24 Potential energy of a magnetic dipole **m** in an external magnetic field **B**.

To go to this Interactive Example click on this sentence.

26.17 Magnetic Flux

Just as the electric flux was defined in chapter 4 as a quantitative measure of the number of electric field lines passing normally through a surface, the magnetic flux Φ_M can be defined as a quantitative measure of the number of magnetic field lines **B** passing normally through a particular surface area **A**. Figure 26.25(a), shows a magnetic field **B** passing through a portion of a surface of area **A**. *The magnetic flux is defined to be*

$$\Phi_M = \mathbf{B} \bullet \mathbf{A} \qquad (26.101)$$

and is a quantitative measure of the number of lines of ***B*** *that pass normally through the surface area* ***A***. The number of lines represents the strength of the field. The

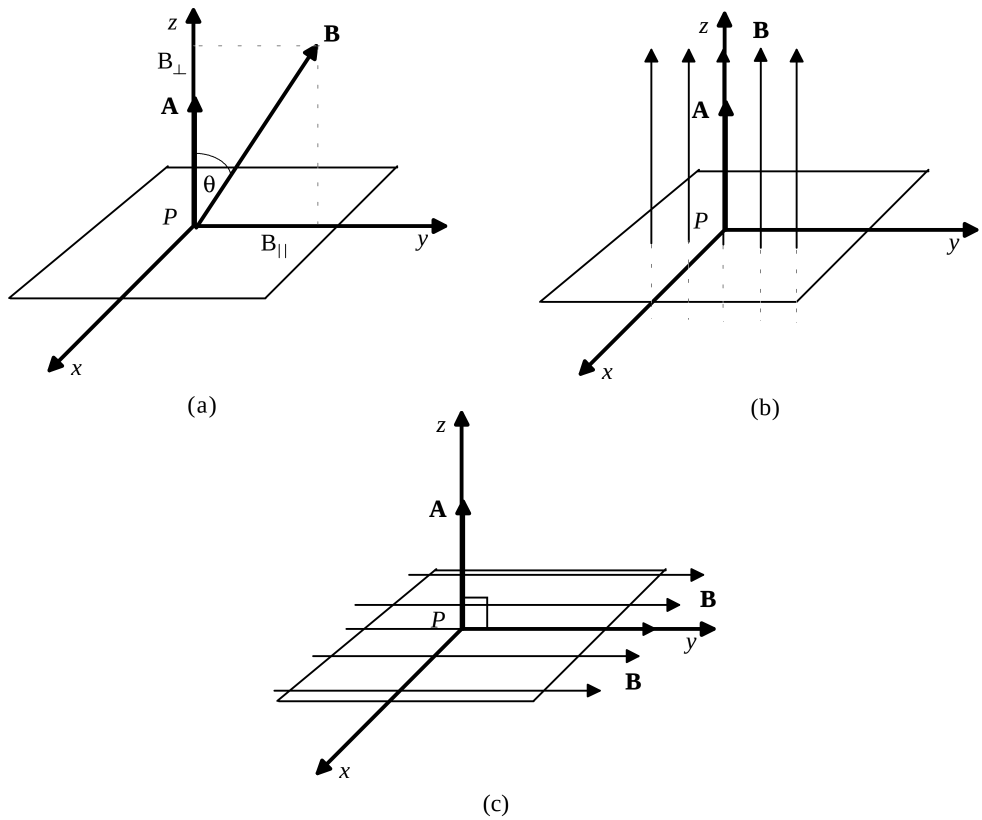


Figure 26.25 The magnetic flux.

vector **B**, at the point *P* of figure 26.25(a), can be resolved into the components, $B_\perp$ the component perpendicular to the surface, and $B_{||}$ the parallel component. The perpendicular component is given by

$$B_\perp = B\cos\theta$$

while the parallel component is given by

$$B_{||} = B\sin\theta$$

The parallel component $B_{||}$ lies in the surface itself and therefore does not pass through the surface, while the perpendicular component $B_\perp$ completely passes through the surface at the point P. The product of the perpendicular component $B_\perp$ and the area A

$$B_\perp A = (B\cos\theta)A = BA\cos\theta = \mathbf{B} \bullet \mathbf{A} = \Phi_M \qquad (26.102)$$

is therefore a quantitative measure of the number of lines of **B** passing normally through the entire surface area **A**. If the angle θ in equation 26.102 is zero, then **B** is parallel to the vector **A** and all the lines of **B** pass normally through the surface area A, as seen in figure 26.25(b). If the angle θ in equation 26.102 is 90^0 then **B** is perpendicular to the area vector **A**, and none of the lines of **B** pass through the surface A as seen in figure 26.25(c).

One of the units for the magnetic field was shown to be 1 Tesla = Weber/m^2. This unit was listed in anticipation of the introduction of the magnetic flux. The unit for magnetic flux will now be defined from the formula as

$$\Phi_M = BA$$

$$\Phi_M = \frac{\text{Weber}}{\text{m}^2}\,\text{m}^2 = \text{Weber}$$

Hence, the unit for magnetic flux is the Weber. It will be abbreviated as Wb.

Example 26.16

Magnetic flux. A magnetic field of 5.00×10^{-2} T passes through a plane 25.0 cm by 35.0 cm at an angle of 40.0^0 to the normal. Find the magnetic flux Φ_M passing through the plane.

Solution

The area of the plane is

$$A = LW = (0.250\text{ m})(0.350\text{ m}) = 8.75 \times 10^{-2}\text{ m}^2$$

The magnetic flux is found from equation 26.102 as

$$\Phi_M = BA\cos\theta$$

$$\Phi_M = (5.00 \times 10^{-2}\text{ T})(8.75 \times 10^{-2}\text{ m}^2)\cos 40.0^0 \left(\frac{\text{Wb/m}^2}{1\text{ T}}\right)$$

$$\Phi_M = 3.35 \times 10^{-3}\text{ Wb}$$

To go to this Interactive Example click on this sentence.

26.18 Gauss's Law for Magnetism

Just as there is an electric flux Φ_E associated with an electric field **E** passing through a surface area **A,** there is also a magnetic flux Φ_M associated with a magnetic field **B** passing through a surface area **A.** The magnetic flux was defined in section 26.17, and was given by equation 26.101 as

$$\Phi_M = \mathbf{B} \bullet \mathbf{A}$$

Because of the similarity of these fluxes it is reasonable to assume that Gauss's law should also apply to magnetism. Gauss's law for electricity was found in equation 22.14 as

$$\Phi_E = \oint \boldsymbol{E} \bullet d\boldsymbol{A} = \frac{q}{\varepsilon_o} \tag{22.14}$$

Where $\Phi_{E,}$ the electric flux was a measure of the electric charge *q* enclosed within the Gaussian surface. It is therefore reasonable to assume that Gauss's law for magnetism should take the same form as equation 22.14. If the assumption is valid, Gauss's law for magnetism should be written as

$$\Phi_{\boldsymbol{M}} = \oint \boldsymbol{B} \bullet d\boldsymbol{A} = \text{enclosed magnetic pole} \tag{26.103}$$

But here we run into a slight difficulty. The magnetic flux should be a measure of the amount of magnetic pole enclosed within the Gaussian surface. *But, as has been seen in section 26.15, isolated magnetic poles do not exist.* Thus the term for the enclosed magnetic pole on the right-hand side of equation 26.103 must be equal to zero. *Thus **Gauss's law for magnetism** becomes*

$$\Phi_{\boldsymbol{M}} = \oint \boldsymbol{B} \bullet d\boldsymbol{A} = 0 \tag{26.104}$$

Just as we considered Gaussian surfaces at various positions in the field of an electric dipole in figure 22.3, we will consider Gaussian surfaces at various positions in the field of a magnetic dipole, represented by the simple bar magnet in figure 26.26.

In a bar magnet, the magnetic field lines also go through the bar, as shown in figure 26.26. *Hence, the amount of magnetic flux entering the Gaussian surface around the north magnetic pole inside the bar magnet is equal to the amount of magnetic flux coming out of the same Gaussian surface outside of the bar magnet. Thus, the flux into the Gaussian surface is equal to the flux out of the Gaussian surface and the net flux through the Gaussian surface surrounding the north magnetic pole is zero.*

In a similar way, the magnetic field lines entering the south magnetic pole continue through the bar magnet until they emerge at the north magnetic pole. *Hence, a Gaussian surface surrounding the south magnetic pole also has a net flux of zero passing through it.* That is, the flux into the Gaussian surface is equal to the

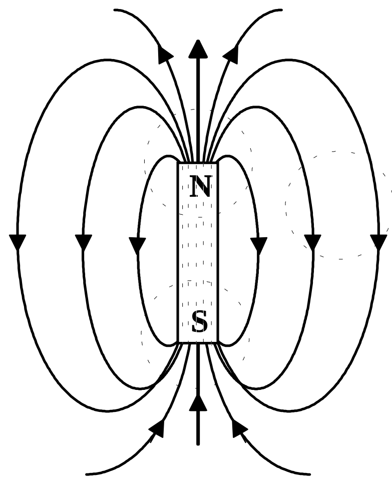


Figure 26.26 Gaussian surfaces and a magnetic dipole.

flux out because all magnetic field lines form closed loops. *The net flux through any Gaussian surface anywhere in a magnetic field is zero because there can never be any isolated magnetic poles.*

The electric field vectors in an electrostatic field always begin and end on electric charges. Because there are no isolated magnetic poles, magnetic field vectors do not begin or end on magnetic poles, but rather, are always continuous.

The Language of Physics

Magnetism
The study of magnetic forces and fields. (p .)

Magnetic Field
The field of force in the neighborhood of a magnetized body, or a current carrying wire. It is measured by the magnetic induction or magnetic flux density. (p .)

Fundamental Principle of Magnetostatics
Like magnetic poles repel each other; unlike magnetic poles attract each other. (p .)

Magnetic Induction or Magnetic flux Density
is equal to the force per unit charge per unit velocity that acts on a charge that is moving perpendicular to the magnetic field. It is also the force acting on a wire of unit length carrying a unit current, when placed in the magnetic field. (p .)

Tesla
The SI unit for the magnetic field. (p .)

Weber
The unit of magnetic flux. (p .)

Right Hand Wire Rule
To determine the direction of the magnetic field around a wire carrying a current, grasp the wire with the right hand, with the thumb in the direction of the current, the fingers will curl around the wire in the direction of the magnetic field. (p .)

Biot-Savart Law
A law that relates the amount of magnetic field generated by a small element of wire carrying a current. (p .)

Ampere's Law
Along any arbitrary path encircling a total current, the sum of the scalar product of the magnetic induction with the element of length of the path, is equal to the permeability times the total current enclosed by the path. (p .)

Magnetic Poles
Since both negative and positive electric charges can exist separately, it is reasonable to ask if magnetic poles can exist separately? The simplest test, would be to cut a bar magnet in half, expecting to obtain one isolated north pole and one isolated south pole. When the experiment is performed, however, two smaller bar magnets are found, each with a north pole and a south pole. No matter how many times the bar magnet is subdivided, an isolated magnetic pole is never found. We always get a dipole. This is why we can never isolate a north pole or a south pole, because they do not exist. What does exist is an orbiting, rotating electric charge, that creates a magnetic dipole moment. (p .)

Magnetic Flux
The magnetic flux is a quantitative measure of the number of lines of the magnetic field **B** that pass normally through a surface area **A**. (p .)

Gauss's law for magnetism
The amount of magnetic flux entering a Gaussian surface is equal to the amount of magnetic flux coming out of the same Gaussian surface. The net flux through any Gaussian surface anywhere in a magnetic field is zero because there are no isolated magnetic poles, and hence, magnetic field vectors do not begin or end on magnetic poles, but rather, are always continuous. (p .)

Summary of Important Equations

Force on a charged particle in an external magnetic field

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \tag{26.1}$$

$$F = qvB \sin\theta \tag{26.2}$$

Definition of the magnetic induction $$B = \frac{F}{qv_\perp} \tag{26.4}$$

Lorentz force $$\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \tag{26.6}$$

Radius of orbit of charged particle in an external magnetic field $$r = \frac{mv}{qB} \tag{26.11}$$

Force on a current carrying conductor in an external magnetic field

$$\mathbf{F} = I\boldsymbol{l} \times \mathbf{B} \tag{26.16}$$

$$F = IlB \sin\theta \tag{26.17}$$

Force on a small portion of a conductor in an external magnetic field

$$d\mathbf{F} = Id\boldsymbol{l} \times \mathbf{B} \tag{26.18}$$

Total force on a conductor $$\boldsymbol{F} = \int d\boldsymbol{F} = \int I\, d\boldsymbol{l} \times \boldsymbol{B} \tag{26.19}$$

Biot-Savart Law $$d\mathbf{B} = \frac{\mu_0 I}{4\pi} \frac{d\boldsymbol{l} \times \mathbf{r}}{r^3} \tag{26.26}$$

Total Magnetic field $$\mathbf{B} = \int d\mathbf{B} \tag{26.27}$$

Magnetic field at the center of a circular current loop $$B = \frac{\mu_0 I}{2r} \tag{26.35}$$

Magnetic field at a point on the z-axis of a circular current loop of radius R carrying a current I $$B = \frac{\mu_0 I R^2}{2(R^2 + z^2)^{\frac{3}{2}}} \tag{26.41}$$

Ampere's circuital law $$\oint \boldsymbol{B} \bullet d\boldsymbol{l} = \mu_0 I \tag{26.42}$$

Magnetic field around a long straight wire $$B = \frac{\mu_0 I}{2\pi r} \tag{26.47}$$

Magnetic field inside a solenoid $$B = \mu_0 n I \tag{26.62}$$

Magnetic field inside a toroid $$B = \frac{\mu_0 N I}{2\pi r} \tag{26.68}$$

Torque on a current loop in an external magnetic field $\tau = IAB\sin\theta$ (26.85)

Magnetic dipole moment of a current loop $\mathbf{m} = IA\,\mathbf{n}$ (26.86)

$\mathbf{m} = NIA\,\mathbf{n}$ (26.87)

Torque on a current loop in an external magnetic field

$$\boldsymbol{\tau} = \mathbf{m} \times \mathbf{B} \quad (26.89)$$

$$\tau = mB\sin\theta \quad (26.88)$$

Potential energy of a magnetic dipole **m** in an external magnetic field **B**

$$\text{PE} = -mB\cos\theta \quad (26.99)$$

$$\text{PE} = -\boldsymbol{m} \bullet \boldsymbol{B} \quad (26.100)$$

Magnetic flux $\Phi_M = \mathbf{B} \bullet \mathbf{A}$ (26.101)

$$\Phi_M = BA\cos\theta \quad (26.102)$$

Gauss's law for magnetism $\Phi_{\boldsymbol{M}} = \oint \boldsymbol{B} \bullet d\boldsymbol{A} = 0$ (26.104)

Questions for Chapter 26

*1. Should there be a law similar to Coulomb's law of electrostatics that shows the force between magnetic poles? What would be the advantages and disadvantages of such a law?

*2. In the very early days of nuclear physics, nuclear radiation was described in terms of alpha, beta, and gamma particles. How did Rutherford use a magnetic field to distinguish between these particles.

3. A charge, in motion with a drift velocity $\mathbf{v}_d$ in a long straight wire, generates a magnetic field around the wire. If you were to move parallel to the wire at the same velocity, would you still observe a magnetic field? If not, where did the field go?

4. Since a moving electric charge creates a magnetic field, does moving a magnet create an electric field?

*5. How can you use a magnetic field to separate isotopes of a chemical element?

6. How can you make a bar magnet?

*7. What causes the earth's magnetic field? Is the field constant or does it change with time? Is it possible for the earth's poles to flip, i.e., for the north pole to become the south and vice versa?

8. If you heat a bar magnet it loses its magnetism. Why?

*9. What is Magneto-hydrodynamics?

Problems for Chapter 26

26.1 The Force on a Charge in a Magnetic Field -- The Definition of the Magnetic Field B

1. A proton, moving at a speed of 1.62×10^3 m/s, enters a magnetic field of 0.250 T at an angle of 43.5^0. Find the force acting on the proton.

2. An electron, moving at a speed of 3.00×10^6 m/s, enters a magnetic field of 0.200 T at an angle of 35.0^0. Find the force acting on the electron.

3. An electron, moving at a speed of 3.00×10^5 m/s, enters a magnetic field of 0.250 T, at an angle of 30.0^0. Find (a) the force on the electron and (b) the acceleration of the electron.

4. What is the force on a proton moving north to south at a speed of 3.00×10^5 m/s in the earth's magnetic field if the vertical component of the earth's magnetic field at that location is 25.0×10^{-6} T?

5. How fast must a proton move in a magnetic field of 2.50×10^{-3} T such that the magnetic force is equal to its weight?

6. Find the value of a magnetic field such that an electron moving at a speed of 2.50×10^4 m/s experiences a maximum force of 2.00×10^{-17} N.

7. An electron is accelerated through a potential difference of 1000 V. It then enters perpendicularly to a uniform magnetic field of 0.200 T. Find the radius of the circular orbit of the electron.

8. Find the value of the magnetic field necessary to cause a proton moving at a speed of 2.50×10^3 m/s to go into a circular orbit of 15.5-cm radius.

9. An electron has an energy of 100 eV as it enters a magnetic field of 3.50×10^{-2} T. Find the radius of the orbit.

10. A velocity selector has a magnetic field of 0.300 T. If a perpendicular electric field of 10,000 V/m is applied, what will be the speed of the particles that will pass through the selector?

11. Find the value of the magnetic field that is necessary for a particle, moving at a speed of 2.5×10^3 m/s, to move straight through an electric field of 500 N/C in a velocity selector.

12. Find the necessary value of the magnetic field in a velocity selector that has an electric field of 500 V/m such that an electron will have an orbital radius of 25.0 cm when it enters a secondary region where the magnetic field is 1.20×10^{-3} T.

26.2 Force on a Current-Carrying Conductor in an External Magnetic Field

13. A wire 35.0 cm long, carrying a current of 3.50 A, is placed at an angle of 40.0^0 in a uniform magnetic field of 0.002 T. Find the force on the wire.

14. What is the maximum force due to the horizontal component of the earth's magnetic field (20.0×10^{-6} T) acting on a 20.0-cm wire, carrying a current of 5.00 A?

15. Find the value of the magnetic field that will cause a maximum force of 7.00×10^{-3} N on a 20.0-cm straight wire carrying a current of 10.0 A.

26.6 The Magnetic Field at the Center of a Circular Current Loop

16. A circular loop of wire of radius 5.00 cm carries a current of 3.00 A. Find the magnetic induction at the center of the current loop.

17. A circular current loop of 10 turns carries a current of 5.00 A. If the radius of the loop is 5.00 cm, find the magnetic field at the center of the loop.

18. It is desired to neutralize the vertical component of the earth's magnetic field (20.0×10^{-6} T) at a particular point. A flat circular coil is mounted horizontally over this point. If the coil has 10 turns and has a radius of 10.0 cm, what current is necessary and in what direction should it flow through the coil?

19. How many loops of wire are necessary to give a magnetic field of 1.50×10^{-3} T at the center of a circular current loop carrying a current of 10.0 A, if the radius of the loop is 5.00 cm?

26.9 Magnetic field around a long straight wire

20. A long straight wire carries a current of 10.0 A. Find the magnetic field 5.00 cm from the wire.

21. A power line 10.0 m high carries a current of 200 A. Find the magnetic field of the wire at the ground.

22. A long straight wire carries a current of 10.0 A. How far from the wire will the magnetic field be (a) 1.00 T, (b) 0.100 T, (c) 1.00×10^{-2} T, and (d) 1.00×10^{-3} T?

23. What current is necessary to generate a magnetic field of 0.100 T at a distance of 10.0 cm from a long straight wire?

24. Find the magnetic field at the position *A* of a long straight wire carrying a current of 10.0 A.

Diagram for problem 24.

25. Two long parallel wires each carry a current of 5.00 A. If the wires are 15.00 cm apart, find the magnetic field midway between them if (a) the currents are in the same direction and (b) the currents are in the opposite direction.

26. Two parallel wires 10.0 cm apart carry currents of 10.0 A each. Find the magnetic field 5.00 cm to the left of wire 1, 5.00 cm to the right of wire 1, and 15.00 cm to the right of wire 1, if (a) the currents are in the same direction and (b) the currents are in the opposite directions.

27. Find the magnetic field at point *A* in the diagram if I_1 (15.0 A) is a current in a wire coming out of the page and I_2 (10.0 A) is a current in a wire going into the page. The distances to point *A* are $r_1 = 5.00$ cm and $r_2 = 10.00$ cm.

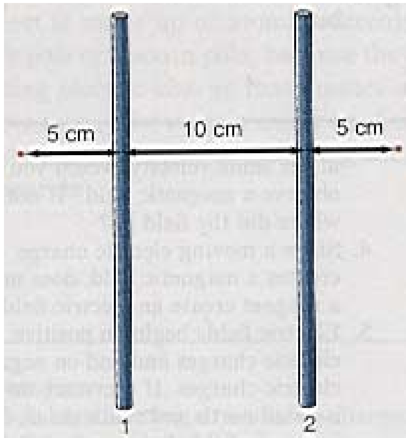


Diagram for problem 26.

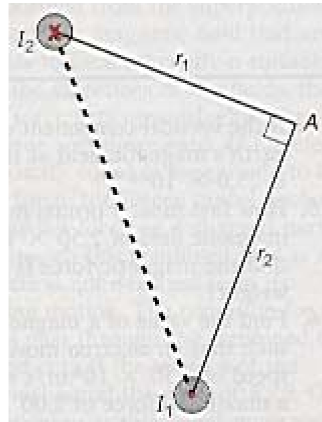


Diagram for problem 27.

28. A solenoid is 20.0 cm long and carries 500 turns of wire. If the current in the solenoid is 2.00 A, find the magnetic field inside the solenoid.

29. You are asked to design a solenoid that will give a magnetic field of 0.100 T, yet the current must not exceed 10.0 A. Find the number of turns per unit length that the solenoid should have.

26.10 Force between Parallel, Current-Carrying Conductors -- The Definition of the Ampere

30. Two parallel wires 15.0 cm apart carry currents of 10.0 A in the same direction. If the wires are 25.0 cm long, (a) find the magnitude and direction of the force on each wire. (b) If the direction of one current is reversed, find the force on each wire.

31. Wire 1 carries a current of 5.00 A; wire 2, 3.00 A; and wire 3, 7.00 A. Find the total force per unit length on wire 2.

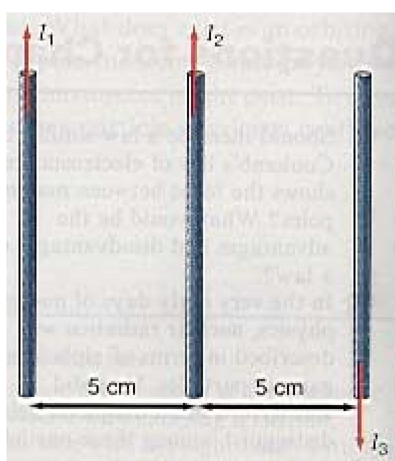


Diagram for problem 31.

32. Two horizontal parallel wires are placed one above the other. Find the equal currents in the two parallel wires, whose repulsive force will just balance the gravitational force on the top wire if the top wire has a mass of 5.00 g, is 20.0 cm long, and is 4.00 cm above the bottom wire.

26.13 Torque on a Current Loop in an External Magnetic Field -- The Magnetic Dipole Moment

33. A coil of wire of 10.0-cm radius carries a current of 5.00 A. Find its magnetic dipole moment.

34. A coil of wire has a magnetic dipole moment of 25.0 A m^2. It is placed perpendicular to the horizontal magnetic field of the earth of 20.0×10^{-6} T. What torque will act on the coil?

35. A coil of wire 30.0 cm in diameter is placed at an angle of 60.0^0 in a magnetic field of 2.50×10^{-2} T and experiences a torque of 3.25×10^{-3} m N. Find (a) the magnetic dipole moment of the coil and (b) the current in the coil.

36. A rectangular galvanometer coil 2.00 cm by 1.50 cm has 10 turns of wire. If the current through the coil is 3.00 mA, find its magnetic dipole moment. The coil is placed in a magnetic field of 0.300 T. Find the torque on the coil when the magnetic dipole moment makes an angle of 30.0^0 with the magnetic field.

37. A coil of 3.00-cm radius, carrying a current of 2.00 A, is placed within a solenoid that carries a current of 3.00 A. If the solenoid has 5000 turns per meter, find the torque on the coil.

*38. A galvanometer coil, 5.00 cm^2 in area, is placed in a magnetic field of 1.00×10^{-3} T. If the coil deflects 40.0^0 when it carries its maximum current of 200 mA, find the torsion constant of the spring in the galvanometer.

Additional Problems

*39. An electron moving at an initial velocity of 6.00×10^5 m/s enters a uniform magnetic field of 2.50×10^{-6} T directed into the paper. The length of the magnetic field is 10.0 cm, as shown in the diagram. Find (a) the force on the electron, (b) the magnitude and direction of the acceleration of the electron, (c) the time that the electron remains in the field, and (d) the amount of deflection of the electron as it leaves the magnetic field.

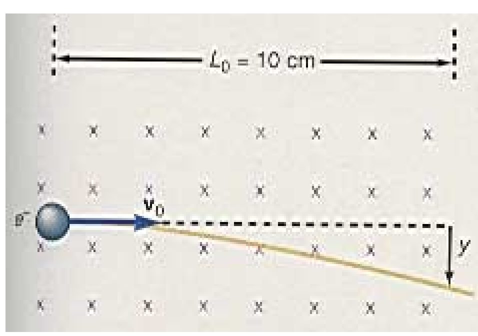


Diagram for problem 39.

40. An electron moves at a speed of 4.00×10^5 m/s parallel to a long straight wire carrying a 12.0-A current. The velocity of the electron is in the same direction as the current in the wire. If the electron is 5.00 cm from the current, find the magnitude and direction of the force on the electron due to the magnetic field of the current.

41. The diagram shows the end view of four wires each carrying a current I of 10.0 A. Wires 1, 2, and 3 have currents coming out of the page, whereas wire 4 has a current going into the page. Find the magnetic field at the center of the square A.

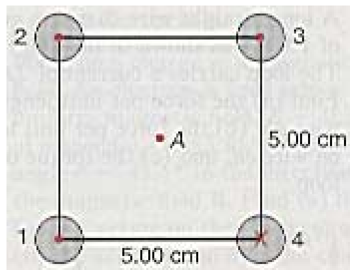


Diagram for problem 41.

42. A 5000-turn solenoid carries a 2.45-A current. A proton is located at the center of the coil, with a velocity of 7.50×10^4 m/s making an angle of 30.0^0 with the axis of the solenoid. Determine the magnitude of the force on the proton due to the magnetic field of the solenoid.

43. A small element of current is located at the center of a flat coil, as shown in the diagram. The coil has 50 turns, it is 50.0 cm in radius, and it carries a current of I = 350 mA in the clockwise direction. The element of current is 2.00 mm long and it carries a current of I = 10.0 mA to the right. Compute the magnitude and direction of the force on the element of current due to the magnetic field of the coil.

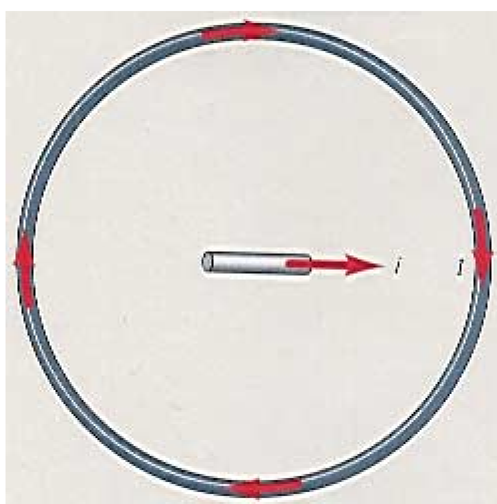


Diagram for problem 43.

44. You are asked to design a circular coil that will have a value of 3.00×10^{-2} T at its center. Find the ratio of the current in the coil to the radius of the coil that will give this value of B. Pick a reasonable value for the combination of I and r such that the current is not too large nor the radius too small. Would it be desirable to introduce more than one loop of wire? What would be a better combination of I, r, and N?

45. An electron, located halfway between two long straight wires, moves at 2.00×10^6 m/s to the left. The top wire carries a 5.00-A current out of the paper, the bottom wire carries a current of 2.00 A into the paper, and the wires are 6.00 cm apart. Find the magnitude and direction of the force on the electron due to the magnetic fields of the current in the wires.

*46. In the Bohr model of the hydrogen atom, a negative electron orbits about a positive proton at a radius of 5.29×10^{-11} m and at a speed of 2.19×10^6 m/s. (a) How long does it take for the electron to revolve around the proton? (b) From the definition of current show that the orbiting electron constitutes a current and determine its magnitude. (c) Since the orbiting electron looks like a current loop, determine the magnetic field at the location of the proton caused by the orbiting electron.

*47. Everyone knows that the earth revolves around the sun. Yet, in our everyday experience the sun is seen to rise in the eastern sky and set in the western sky, as though the sun revolved around the earth. The observed motion seems to depend on the frame of reference of the observer. When the frame of reference is placed on the earth, the sun appears to revolve around the earth. In a similar way, if the frame of reference is placed on the orbiting electron in the Bohr theory of the atom, it appears as though the proton is moving in a circular orbit about the electron. Find the value of the magnetic field at the position of the electron, caused by the proton.

48. Find the magnetic fields at the center of the following two coils: (a) a 100-turn coil 20.0 cm long with 0.200-cm diameter, carrying a current of 2.00 A, and (b) a 100-turn coil 20.0 cm in diameter and 0.200 cm thick, carrying a current of 2.00 A.

*49. You are asked to design a solenoid by wrapping insulated copper wire around the hollow cardboard core of an empty roll of paper towels. The completed solenoid will then be connected to a 12.0-V battery and the maximum current that can flow in the circuit is 10.0 A. (a) What is the minimum value of the resistance of this solenoid? (b) If the wire used is #22 S&W gauge copper wire, which has a diameter of 6.44×10^{-4} m, what is the minimum length of this wire? (c) With the above restrictions on I and l, how many turns of wire can you have if the diameter of the solenoid is 4.00 cm? (d) If the length of the cardboard core is 28.0 cm, and with all of the above restrictions, find the value of B inside the solenoid. (e) In this design, what factors might be changed to increase the value of B?

*50. A solenoid is filled with iron, which has a permeability of $\mu = 4.80 \times 10^{-3}$ T m/A. The solenoid has 200 turns of wire, is 30.0 cm long, and carries a current of

5.00 A. Find the value of B inside the solenoid. *(Hint:* just as the electric field in a vacuum was characterized by the permittivity of free space ε_0, and an electric field in a different medium was characterized by ε, the permittivity of the medium the magnetic field in a vacuum is characterized by the permeability μ_0, while the magnetic field in a medium is characterized by the permeability of that medium μ.)

*51. You are to design a galvanometer that has an internal resistance R_g = 20.0 Ω and gives full-scale deflection for a current I_g = 20.0 mA. (a) Find the length of #22 B&S gauge copper wire necessary to give this resistance. (b) If this length of wire is to be made into a 500 turn loop, find the radius of the loop. (c) Find the area A of this loop. (d) Find the magnetic dipole moment of this loop. (e) If this loop is placed in a magnetic field of 2.50×10^{-2} T, find the maximum torque on the coil. (f) If the galvanometer needle is to rotate through an angle of $\pi/2$ rad, find the torsion constant of the spring that will supply the countertorque.

52. A long straight wire carries a current of 10.0 A as shown in the diagram. The loop carries a current of 2.00 A. Find (a) the force per unit length on wire *ab*, (b) the force per unit length on wire *cd*, and (c) the torque on the loop.

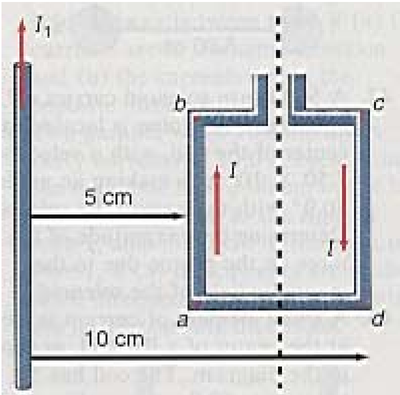


Diagram for problem 52.

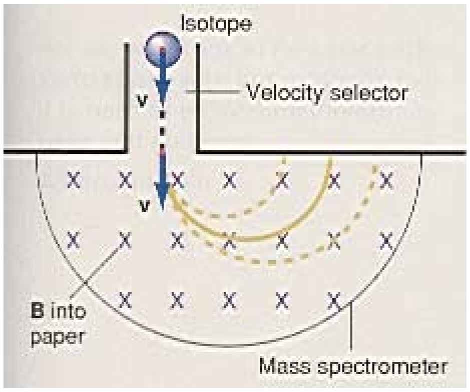


Diagram for problem 54.

*53. The coil in a DC motor is called an *armature.* The armature, 15.0 cm by 10.0 cm, with 100 turns of wire, carries a current of 3.50 A in a magnetic field of 0.250 T. Find the maximum torque exerted by the motor. If the armature rotates at 1000 rpm, what is the power rating of the motor?

54. Every chemical element has isotopes. An *isotope* has the same number of protons as the element but a different number of neutrons. Hence an isotope has a different mass than the parent element. The different masses of the isotopes can be found with a mass spectrometer, as shown in the diagram. Ions of the isotope are fired into the velocity selector, and those of velocity v enter into the mass spectrometer where there is a magnetic field B into the paper. The path of the ions is bent into a circular path of radius R. (a) Find the equation for the mass of any isotope. (b) The chemical element oxygen has eight protons and eight neutrons. It has two stable isotopes, one with nine neutrons and the other with ten neutrons.

Find the radii of the path of the isotopes in the spectrometer in terms of the radius of the oxygen element.

Interactive Tutorials

55. *Force on a charge in a magnetic field.* An electron is fired into a uniform magnetic field, B = 0.255 T, at a speed v = 185 m/s and at an angle $\theta = 43.5^0$ to the direction of the magnetic field **B.** Find (a) the force F acting on the charge q and (b) the acceleration a of the charge.

56. *Force on a current-carrying wire in a magnetic field.* A length of wire L = 42.0 cm carries a current I = 15.0 A. The wire is placed in a uniform magnetic field, B = 0.365 T. If the wire makes an angle $\theta = 55.5^0$ with the direction of the magnetic field **B,** find the magnitude of the force **F** on the wire.

57. *The magnetic field at the center of a circular current loop.* Find the magnetic field at the center of a circular current loop of N = 10 turns with a radius r = 5.00 cm that carries a current I = 10.0 A.

58. *The magnetic field of a long straight wire.* Find the magnetic field at a distance r = 10.0 cm from a long straight wire that carries a current I = 12.5 A.

59. *The magnetic field inside a solenoid.* Find the magnetic field inside a solenoid that has a length L = 25.0 cm and is composed of N = 1000 turns of wire. The current in the solenoid is I = 7.50 A.

60. *The torque on a current loop in an external magnetic field.* A circular coil, consisting of N = 15 turns of wire, having a diameter d = 12.5 cm, carries a current I = 8.00 A. The coil is placed in a uniform magnetic field B = 0.385 T. Find (a) the magnetic dipole moment m of the coil and (b) the maximum torque $\tau_{max.}$ (c) Plot the torque acting on the coil as a function of the angle θ that the magnetic dipole moment of the coil makes with the magnetic field.

61. *Magnetic flux.* A uniform magnetic field, $B = 3.55 \times 10^{-2}$ T, passes through a plane surface of area $A = 9.35 \times 10^{-2}$ m^2 at an angle $\theta = 53.5^0$ to the normal of the surface. Find the magnetic flux Φ_M passing through the plane surface.

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