

Chapter 21 Electric Fields

21.1 The Electric Field

In chapter 20, Coulomb's law of electrostatics was discussed and it was shown that, if a charge q_2 is brought into the neighborhood of another charge q_1 , a force is exerted upon q_2 . The magnitude of that force is given by Coulomb's law. However, it should be asked what is the mechanism that transmits the force from q_1 to q_2 ? Coulomb's law states only that there is a force; it says nothing about the mechanism by which the force is transmitted, and it assumes that the force is transmitted instantaneously. Such a force is called an "action at a distance" since it does not explain how the force travels through that distance. Even the ancient Greek Philosophers would not have accepted such an idea, it seems too much like magic.

To overcome this shortcoming of Coulomb's law, Michael Faraday (1791-1867) introduced the concept of an electric field. He stated that *it is an intrinsic property of nature that an electric field exists in the space around an electric charge. This electric field is considered to be a force field that exerts a force on charges placed in the field.* As an example, around the charge q_1 there exists an electric field. When the charge q_2 is brought into the neighborhood of q_1 , the electric field of q_1 interacts with q_2 , thereby exerting a force on q_2 . The electric field becomes the mechanism for transmitting the force from q_1 to q_2 , thereby eliminating the "action at a distance" principle.

Because the electric field is considered to be a force field, the existence of an electric field and its strength is determined by the effect it produces on a positive point charge q_0 placed in the region where the existence of the field is suspected. If the point charge, called a test charge, experiences an electrical force acting upon it, then it is said that the test charge is in an electric field. *The electric field is measured in terms of a quantity called the electric field intensity. The magnitude of the electric field intensity is defined as the ratio of the force F acting on the small test charge, q_0 , to the small test charge itself. The direction of the electric field is in the direction of the force on the positive test charge.* This can be written as

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} \quad (21.1)$$

i.e., the force acting per unit charge. The SI unit of electric field intensity is a newton per coulomb, abbreviated as N/C. It should also be noted at this point that the small positive test charge q_0 must be small enough so that it will not appreciably distort the electric field that you are trying to measure. (In the measurement of any physical quantity the instruments of measurement should be designed to interfere as little as possible with the quantity being measured.) To emphasize this point equation 21.1 is sometimes written in the form

$$\mathbf{E} = \lim_{q_0 \rightarrow 0} \frac{\mathbf{F}}{q_0}$$

21.2 The Electric Field of A Point Charge

The electric field of a positive point charge q can be determined by following the definition in equation 21.1. A positive point charge q is shown in figure 21.1. A very small positive test charge q_0 is placed in various positions around the positive

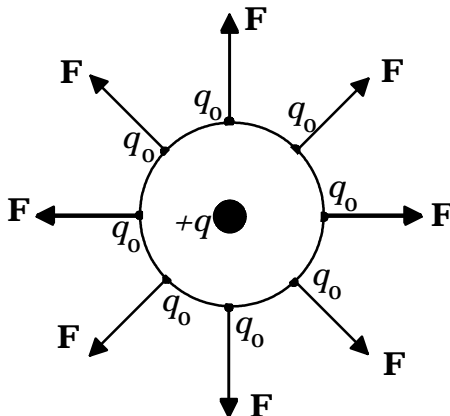


Figure 21.1 The force exerted by a positive point charge.

charge q . Because like charges repel each other, the positive test charge q_0 will experience a force of repulsion from the positive point charge q . Thus, the force acting on the test charge, and hence the direction of the electric field, is always directed radially away from the point charge q . The magnitude of the electric field intensity of a point charge is found from equation 21.1, with the force found from Coulomb's law, equation 20.1.

$$F = k \frac{qq_0}{r^2} \quad (20.1)$$

That is,

$$E = \frac{F}{q_0} = \frac{k \frac{qq_0}{r^2}}{q_0}$$

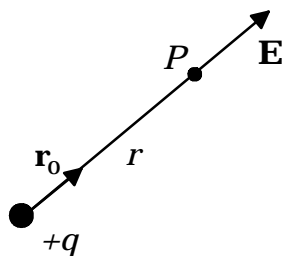
$$E = k \frac{q}{r^2} \quad (21.2)$$

Equation 21.2 is the equation for the magnitude of the electric field intensity due to a point charge. The direction of the electric field has already been shown to be radially away from the positive point charge. If a *unit vector* \mathbf{r}_o is drawn pointing everywhere radially away from the point charge then equation 21.2 can be written in the vector form

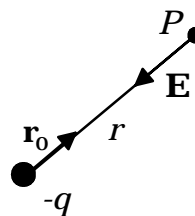
$$\mathbf{E} = k \frac{q}{r^2} \mathbf{r}_o \quad (21.3)$$

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As an example, the electric field \mathbf{E} at an arbitrary point P , figure 21.2(a), points radially outward if q is positive, and radially inward if q is negative, figure 21.2(b).



(a) Positive Charge

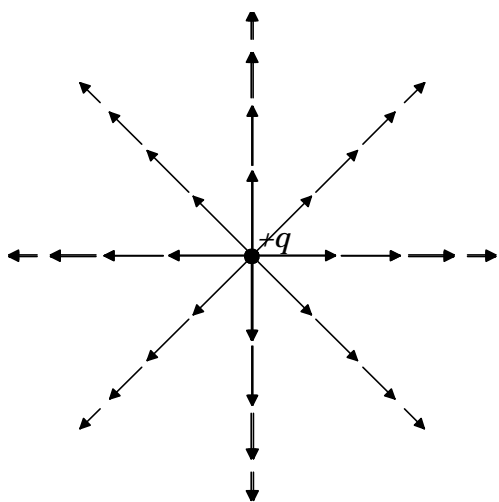


(b) Negative Charge

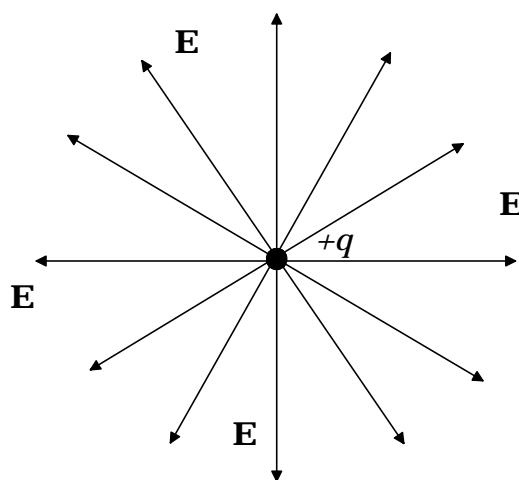
Figure 21.2 The direction of the unit vector \mathbf{r}_0 and the electric field vector \mathbf{E} .

To draw a picture of the total electric field of a positive point charge is slightly difficult because the electric field is a vector, and as you recall from the study of vectors, a vector is a quantity that has both magnitude and direction. Since the magnitude of the electric field of the point charge varies with the distance r from the charge, a picture of the total electric field would have to be a series of discrete electric vectors each one pointing radially away from the positive point charge but the length of each vector would vary depending upon how far you are away from the point charge. This is shown in figure 21.3(a).

To simplify the picture of the electric field, a series of continuous lines are drawn from the positive point charge to indicate the total electric field, as in figure 21.3(b). These continuous lines were called by Michael Faraday, lines of force because they are in the direction of the force that acts upon a positive point charge



(a)



(b)

Figure 21.3 Electric field of a positive point charge.

placed in the field. They are everywhere tangent to the direction of the electric field. It must be understood however, that the magnitude of the electric field varies along the lines shown. The greater the distance from the point charge the smaller the magnitude of the electric field. With these qualifications in mind, we will say that the electric field of a positive point charge is shown in figure 21.3(b). (We will use this same technique to depict the electric fields in the rest of this book.) Note that the electric field always emanates from a positive charge. *The **electric field intensity of a point charge** is directly proportional to the charge that creates it, and inversely proportional to the square of the distance from the point charge to the position where the field is being evaluated.*

The electric field intensity of a negative point charge is found in the same way. But because unlike charges attract each other, the force between the negatively charged point source and the small positive test charge is one of attraction. Hence, the force is everywhere radially inward toward the negatively charged point source and the electric field is also. Therefore, the electric field of a negative point charge is as shown in figure 21.4. The magnitude of the electric field intensity of a negative point charge is also given by equation 21.2.

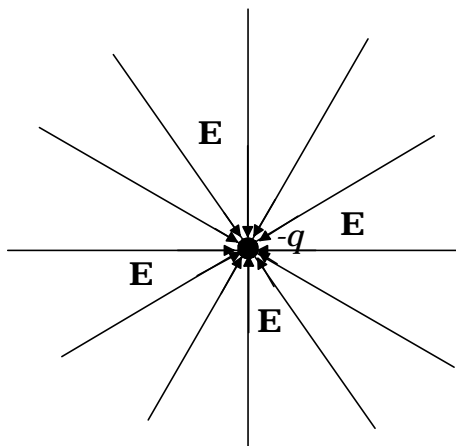


Figure 21.4 Electric field of a negative point charge.

Example 21.1

The electric field of a point charge. Find the magnitude of the electric field intensity at a distance of 0.500 m from a 3.00 μC charge.

Solution

The electric field intensity is found from equation 21.2 as

$$E = k \frac{q}{r^2}$$

$$E = (9.00 \times 10^9 \text{ N m}^2/\text{C}^2) \frac{(3.00 \times 10^{-6} \text{ C})}{(5.00 \times 10^{-1} \text{ m})^2}$$

$$E = 1.08 \times 10^5 \text{ N/C}$$

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If the electric field intensity is known in a region, then the force on any charge q placed in that field is determined from equation 21.1 as

$$\mathbf{F} = q\mathbf{E} \quad (21.4)$$

Example 21.2

The force on a charge in an electric field. A point charge, $q = 5.64 \mu\text{C}$, is placed in an electric field of $2.55 \times 10^3 \text{ N/C}$. Find the magnitude of the force acting on the charge.

Solution

The magnitude of the force acting on the point charge is found from equation 21.4 as

$$F = qE$$

$$F = (5.64 \times 10^{-6} \text{ C})(2.55 \times 10^3 \text{ N/C})$$

$$F = 1.44 \times 10^{-2} \text{ N}$$

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21.3 Superposition of Electric Fields for Multiple Discrete Charges

When more than one charge is present, as in figure 21.5, the force on an arbitrary charge q is seen to be the vector sum of the forces produced by each charge, i.e.,

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \dots \quad (21.5)$$

But if charge q_1 produces a field \mathbf{E}_1 , then the force on charge q produced by \mathbf{E}_1 , is found from equation 21.4 as

$$\mathbf{F}_1 = q\mathbf{E}_1 \quad (21.6)$$

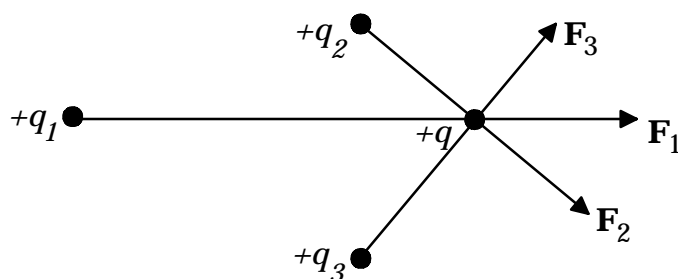


Figure 21.5 Multiple charges.

Similarly, the force on charge q produced by \mathbf{E}_2 is

$$\mathbf{F}_2 = q\mathbf{E}_2 \quad (21.7)$$

and finally

$$\mathbf{F}_3 = q\mathbf{E}_3 \quad (21.8)$$

Substituting equations 21.6, 21.7 and 21.8 into 21.5 gives

$$\mathbf{F} = q\mathbf{E}_1 + q\mathbf{E}_2 + q\mathbf{E}_3 + \dots$$

Dividing each term by q gives

$$\mathbf{F}/q = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \dots \quad (21.9)$$

But \mathbf{F}/q is the resultant force per unit charge acting on charge q and is thus the total resultant electric field intensity \mathbf{E} . Therefore equation 21.9 becomes

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \dots \quad (21.10)$$

Equation 21.10 is the mathematical statement of the principle of the superposition of electric fields: When more than one charge contributes to the electric field, the resultant electric field is the vector sum of the electric fields produced by the various charges.

Example 21.3

The electric field of two positive charges. If two equal positive charges, $q_1 = q_2 = 2.00 \mu\text{C}$ are situated as shown, in figure 21.6, find the resultant electric field intensity at point A. The distance $r_1 = 0.819 \text{ m}$, $r_2 = 0.574 \text{ m}$, and $l = 1.00 \text{ m}$.

Solution

The magnitude of the electric field intensity produced by q_1 is found from equation 21.2 as

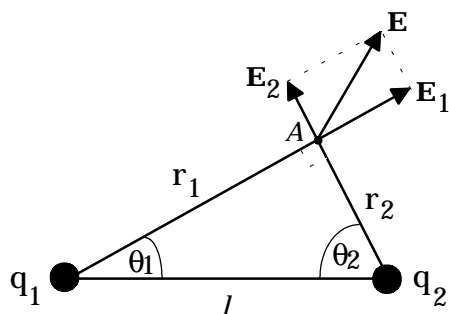


Figure 21.6 The electric field of two positive charges.

$$E_1 = k \frac{q_1}{r_1^2}$$

$$E_1 = (9.00 \times 10^9 \text{ N m}^2/\text{C}^2) \frac{(2.00 \times 10^{-6} \text{ C})}{(0.819 \text{ m})^2}$$

$$E_1 = 2.68 \times 10^4 \text{ N/C}$$

The magnitude of the electric field intensity produced by q_2 is

$$E_2 = k \frac{q_2}{r_2^2}$$

$$E_2 = (9.00 \times 10^9 \text{ N m}^2/\text{C}^2) \frac{(2.00 \times 10^{-6} \text{ C})}{(0.574 \text{ m})^2}$$

$$E_2 = 5.46 \times 10^4 \text{ N/C}$$

The resultant electric field is found from equation 21.10 as the vector addition

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

where

$$\mathbf{E}_1 = \mathbf{i}E_{1x} + \mathbf{j}E_{1y}$$

and

$$\mathbf{E}_2 = -\mathbf{i}E_{2x} + \mathbf{j}E_{2y}$$

as can be seen in figure 21.7. Hence the resultant vector is

$$\mathbf{E} = \mathbf{i}E_{1x} + \mathbf{j}E_{1y} - \mathbf{i}E_{2x} + \mathbf{j}E_{2y}$$

$$\mathbf{E} = \mathbf{i}(E_{1x} - E_{2x}) + \mathbf{j}(E_{1y} + E_{2y})$$

Therefore,

$$E_x = E_{1x} - E_{2x}$$

and

$$E_y = E_{1y} + E_{2y}$$

The angles θ_1 and θ_2 are found from figure 21.6 as follows

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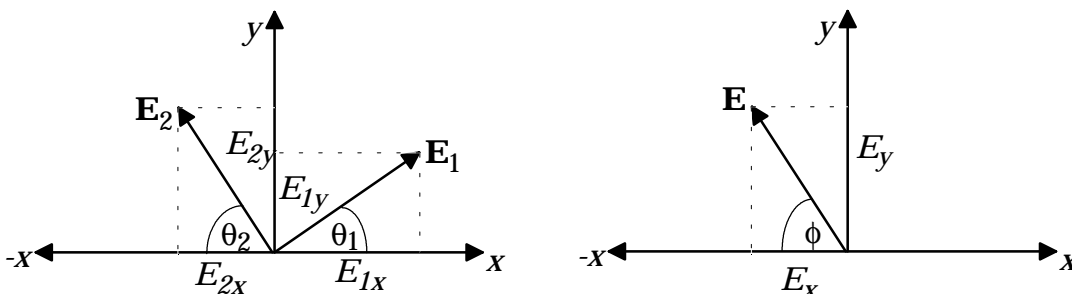


Figure 21.7 The components of the electric field vectors.

$$\theta_1 = \tan^{-1}\left(\frac{r_2}{r_1}\right) = \tan^{-1}\left(\frac{0.574 \text{ m}}{0.819 \text{ m}}\right) = 35.0^\circ$$

$$\theta_2 = 90.0^\circ - \theta_1 = 90.0^\circ - 35.0^\circ = 55.0^\circ$$

The x -components of the electric field intensities are

$$E_{1x} = E_1 \cos\theta_1$$

$$E_{1x} = (2.68 \times 10^4 \text{ N/C})\cos 35.0^\circ$$

$$E_{1x} = 2.20 \times 10^4 \text{ N/C}$$

and

$$E_{2x} = E_2 \cos\theta_2$$

$$E_{2x} = (5.46 \times 10^4 \text{ N/C})\cos 55.0^\circ$$

$$E_{2x} = 3.13 \times 10^4 \text{ N/C}$$

Hence the x -component of the resultant field is

$$E_x = E_{1x} - E_{2x}$$

$$E_x = 2.20 \times 10^4 \text{ N/C} - 3.13 \times 10^4 \text{ N/C}$$

$$E_x = -0.930 \times 10^4 \text{ N/C}$$

The y -components of the electric field intensities are

$$E_{1y} = E_1 \sin\theta_1$$

$$E_{1y} = (2.68 \times 10^4 \text{ N/C})\sin 35.0^\circ$$

$$E_{1y} = 1.54 \times 10^4 \text{ N/C}$$

and

$$E_{2y} = E_2 \sin\theta_2$$

$$E_{2y} = (5.46 \times 10^4 \text{ N/C})\sin 55.0^\circ$$

$$E_{2y} = 4.47 \times 10^4 \text{ N/C}$$

Therefore the y -component of the resultant field is

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$$\begin{aligned}E_y &= E_{1y} + E_{2y} \\E_y &= 1.54 \times 10^4 \text{ N/C} + 4.47 \times 10^4 \text{ N/C} \\E_y &= 6.01 \times 10^4 \text{ N/C}\end{aligned}$$

The resultant electric field vector is given by equation 21.11 as

$$\begin{aligned}\mathbf{E} &= \mathbf{i}E_x + \mathbf{j}E_y \\ \mathbf{E} &= - (0.930 \times 10^4 \text{ N/C})\mathbf{i} + (6.01 \times 10^4 \text{ N/C})\mathbf{j}\end{aligned}\tag{21.11}$$

The magnitude of the resultant electric field intensity at point *A* is found as

$$\begin{aligned}E &= \sqrt{E_x^2 + E_y^2} \\ E &= \sqrt{(-0.930 \times 10^4 \text{ N/C})^2 + (6.01 \times 10^4 \text{ N/C})^2} \\ E &= 6.08 \times 10^4 \text{ N/C}\end{aligned}\tag{21.12}$$

The direction of the electric field vector is found from

$$\begin{aligned}\tan \phi &= \frac{E_y}{E_x} \\ \phi &= \tan^{-1} \frac{E_y}{E_x} = \tan^{-1} \left(\frac{6.01 \times 10^4 \text{ N/C}}{-0.930 \times 10^4 \text{ N/C}} \right) = \tan^{-1}(-6.46) \\ \phi &= -81.2^\circ\end{aligned}$$

Because E_x is negative, the angle ϕ lies in the second quadrant. The angle that the vector \mathbf{E} makes with the positive x -axis is $\phi + 180^\circ = 98.8^\circ$.

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Thus, by the principle of superposition, the electric field can be determined at any point for any number of charges. However, we have only found the field at one point. If it is desired to see a picture of the entire electric field, as already shown for the point charge, \mathbf{E} must be evaluated vectorially at an extremely large number of points. As can be seen from this example this would be a rather lengthy job. However, the entire electric field can be readily solved by the use of a computer. The total electric field caused by two equal positive charges, q_1 and q_2 , is shown in figure 21.8.

Equation 21.10 gives the electric field for any number of discrete charges. To find the electric field of a continuous distribution of charge, it is necessary to use the calculus and the sum in equation 21.10 becomes an integration. We will treat continuous charge distributions in section 21.6.

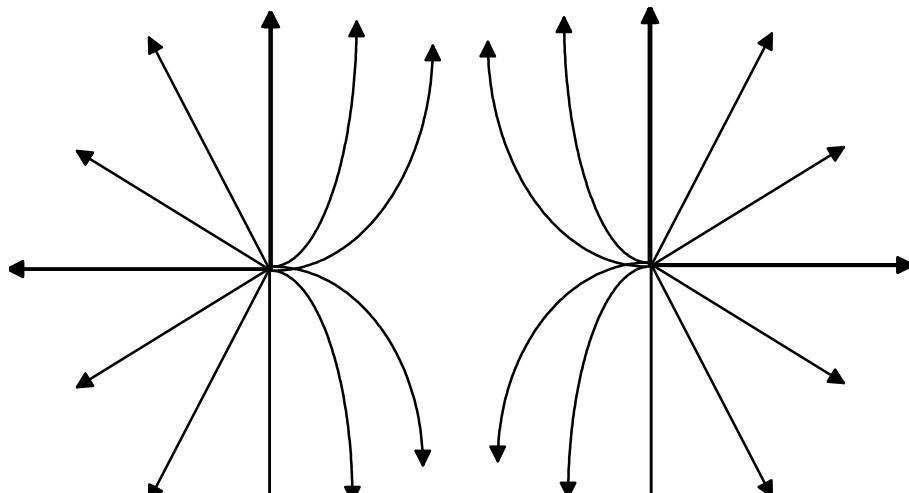


Figure 21.8 Electric field of two equal positively charged particles.

21.4 The Electric Field along the Perpendicular Bisector of an Electric Dipole

The configuration of two closely spaced, equal but opposite point charges is a very important one and is given the special name of an electric dipole. The electric field intensity at a point P along the perpendicular bisector of an electric dipole can be found with the help of figure 21.9.

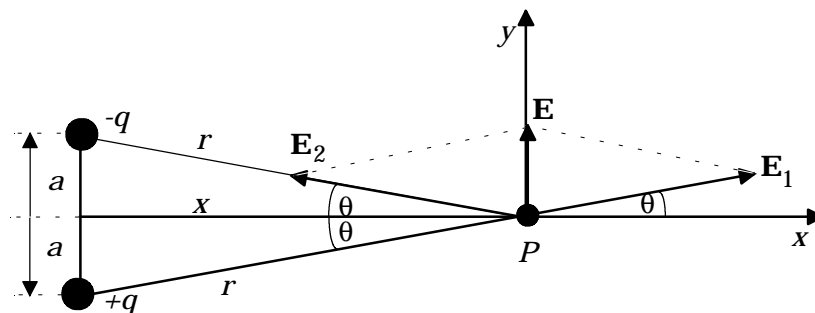


Figure 21.9 The electric dipole.

The resultant electric field intensity is found by the superposition principle as

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$

The magnitudes of \mathbf{E}_1 and \mathbf{E}_2 are found as

$$E_1 = k\frac{q}{r^2} = E_2 = k\frac{q}{r^2}$$

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As can be seen in figure 21.9, the electric fields are written in terms of their components as

$$\mathbf{E}_1 = \mathbf{i}E_{1x} + \mathbf{j}E_{1y}$$

and

$$\mathbf{E}_2 = -\mathbf{i}E_{2x} + \mathbf{j}E_{2y}$$

Thus the resultant electric field in terms of its components is

$$\begin{aligned}\mathbf{E} &= \mathbf{i}E_{1x} + \mathbf{j}E_{1y} - \mathbf{i}E_{2x} + \mathbf{j}E_{2y} \\ \mathbf{E} &= \mathbf{i}(E_{1x} - E_{2x}) + \mathbf{j}(E_{1y} + E_{2y})\end{aligned}$$

Because $E_1 = E_2$, the x -component of the resultant field at the point P becomes

$$\begin{aligned}E_x &= E_{1x} - E_{2x} \\ E_x &= E_1 \cos\theta - E_2 \cos\theta \\ E_x &= E_1 \cos\theta - E_1 \cos\theta \\ E_x &= 0\end{aligned}$$

Also because $E_1 = E_2$, the y -component of the resultant field at the point P is

$$\begin{aligned}E_y &= E_{1y} + E_{2y} \\ E_y &= E_1 \sin\theta + E_2 \sin\theta \\ E_y &= E_1 \sin\theta + E_1 \sin\theta \\ E_y &= 2E_1 \sin\theta\end{aligned}$$

The resultant electric field is therefore

$$\begin{aligned}\mathbf{E} &= \mathbf{i}E_x + \mathbf{j}E_y \\ \mathbf{E} &= \mathbf{i}(0) + \mathbf{j}(2E_1 \sin\theta)\end{aligned}$$

Therefore the resultant electric field is

$$\mathbf{E} = 2E_1 \sin\theta \mathbf{j}$$

But $E_1 = kq/r^2$, hence

$$\mathbf{E} = \frac{2kq}{r^2} \sin\theta \mathbf{j} \tag{21.13}$$

But from figure 21.9 it is seen that

$$\sin\theta = a/r$$

Therefore,

$$\begin{aligned}\mathbf{E} &= \frac{2kq}{r^2} \frac{a}{r} \mathbf{j} \\ \mathbf{E} &= \frac{k2aq}{r^3} \mathbf{j}\end{aligned}$$

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However, from the diagram it is seen that

$$r = \sqrt{a^2 + x^2}$$

Thus,

$$\mathbf{E} = \frac{k2aq}{(\sqrt{a^2 + x^2})^3} \mathbf{j} = \frac{k2aq}{(a^2 + x^2)^{3/2}} \mathbf{j} \quad (21.14)$$

The quantity $(2aq)$ which is the product of the charge q times the distance separating the charges, $2a$, is called the electric dipole moment and is designated by the letter p . That is,

$$p = 2aq \quad (21.15)$$

The SI unit for the electric dipole is a coulomb meter, abbreviated C m. In terms of the electric dipole moment p , the electric field of a dipole along its perpendicular bisector is given by

$$\mathbf{E} = \frac{kp}{(a^2 + x^2)^{3/2}} \mathbf{j} \quad (21.16)$$

In practice x is usually very much greater than a , that is $x \gg a$, so as a first approximation we can let

$$(a^2 + x^2)^{3/2} \approx (0 + x^2)^{3/2} = x^3$$

Therefore, the magnitude of the electric field intensity along the perpendicular bisector, a distance x from the dipole, is

$$\mathbf{E} = \frac{kp}{x^3} \mathbf{j} \quad (21.17)$$

The direction of the electric field along the perpendicular bisector of the electric dipole is in the $+\mathbf{j}$ direction as can be seen from figure 21.9 and is parallel to the axis of the dipole. Note that the electric field of the dipole varies as $1/x^3$, while the electric field of a point charge varies as $1/x^2$. Thus the electric field of a dipole decreases faster with distance than the electric field of a point charge.

We have found the electric field at only one point in space. If it is desired to see a picture of the entire electric field of the dipole, \mathbf{E} must be evaluated vectorially at an extremely large number of points. As can be seen from this example this would be a rather lengthy job. However, the entire electric field can be readily solved by the use of a computer. The total electric field of an electric dipole is shown in figure 21.10. More detail on the electric dipole will be found in sections 21.5, 23.6 and 23.7.

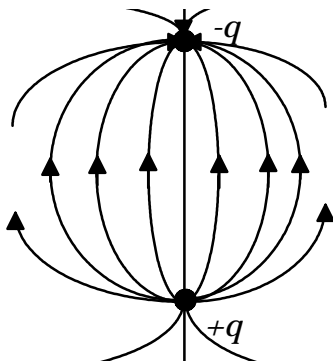


Figure 21.10 The electric field of a dipole.

Example 21.4

Electric field of a dipole. An electric dipole consists of two charges, $q_1 = 2.00 \mu\text{C}$ and $q_2 = -2.00 \mu\text{C}$ separated by a distance of 5.00 mm. Find (a) the electric dipole moment and (b) the resultant electric field intensity at a point on the perpendicular bisector of 1.50 m from the center of the dipole.

Solution

a. the electric dipole moment is found from equation 21.15 as

$$p = 2aq$$

$$p = (5.00 \times 10^{-3} \text{ m})(2.00 \times 10^{-6} \text{ C})$$

$$p = 1.00 \times 10^{-8} \text{ m C}$$

b. The magnitude of the electric field intensity of the dipole is found from equation 21.16 as

$$\mathbf{E} = \frac{kp}{(a^2 + x^2)^{3/2}} \mathbf{j}$$

$$\mathbf{E} = \frac{(9.00 \times 10^9 \text{ N m}^2/\text{C}^2)(1.00 \times 10^{-8} \text{ m C})}{[(2.50 \times 10^{-3} \text{ m})^2 + (1.50 \text{ m})^2]^{3/2}} \mathbf{j}$$

$$\mathbf{E} = (26.7 \text{ N/C}) \mathbf{j}$$

Notice that the same answer would have been obtained if the quantity “ a ” was set equal to zero, since $1.50 \text{ m} \gg 2.50 \times 10^{-3} \text{ m}$, which is consistent with the usual assumption that $x \gg a$.

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21.5 The Torque on a Dipole in an External Electric Field

As shown in the last section, the electric dipole moment was defined by equation 21.15 as

$$p = 2aq$$

The electric dipole moment can be written as a vector by defining a unit vector \mathbf{r}_o that points from the negative charge to the positive charge. The electric dipole moment can then be written as

$$\mathbf{p} = 2aq\mathbf{r}_o \quad (21.18)$$

The electric dipole moment thus points from the negative charge to the positive charge as seen in figure 21.11a.

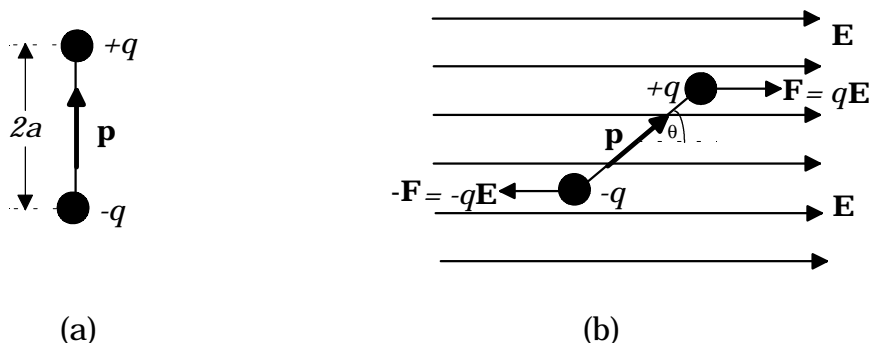


Figure 21.11 Torque on a dipole in an external electric field.

Let us now place this electric dipole into the external electric field shown in figure 21.11b. The charge $+q$ experiences the force

$$\mathbf{F} = q\mathbf{E} \quad (21.19)$$

to the right as shown, while the charge $-q$ experiences the force

$$-\mathbf{F} = -q\mathbf{E} \quad (21.20)$$

to the left as shown. Hence the net force on the dipole is zero. That is, the dipole will not accelerate in the x or y -direction. However, because the forces do not act through the same point, they will produce a torque on the dipole. The torque $\boldsymbol{\tau}$ acting on the dipole is given by

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F} + (-\mathbf{r}) \times (-\mathbf{F}) = 2\mathbf{r} \times \mathbf{F} \quad (21.21)$$

where \mathbf{r} is the vector distance from the center of the dipole to the charge $+q$ and $-\mathbf{r}$ is the vector distance from the center of the dipole to the charge $-q$. The distance $2r$

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is thus equal to $2a$ the separation between the charges and hence $2\mathbf{r}$ can be written as

$$2\mathbf{r} = 2a\mathbf{r}_0 \quad (21.22)$$

where \mathbf{r}_0 is a unit vector pointing from the negative charge to the positive charge. Replacing equations 21.19 and 21.22 into 21.21 gives

$$\begin{aligned} \boldsymbol{\tau} &= 2\mathbf{r} \times \mathbf{F} = (2a\mathbf{r}_0) \times (q\mathbf{E}) \\ \boldsymbol{\tau} &= (2aq\mathbf{r}_0) \times (\mathbf{E}) \end{aligned} \quad (21.23)$$

But $(2aq\mathbf{r}_0)$ is equal to the electric dipole moment \mathbf{p} defined in equation 21.18. Replacing equation 21.18 into equation 21.23 gives for the torque acting on an electric dipole in an external field

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} \quad (21.24)$$

The magnitude of the torque is given by the definition of the cross product as

$$\tau = pE \sin\theta \quad (21.25)$$

When p is perpendicular to the external electric field E , θ is equal to 90° , and the $\sin 90^\circ = 1$. Therefore the torque acting on the dipole will be at its maximum value, and will act to rotate the dipole clockwise in figure 21.11b. As the dipole rotates, the angle θ will decrease until the electric dipole moment becomes parallel to the external electric field, and the angle θ will be equal to zero. At this point the torque acting on the dipole becomes zero, because the $\sin\theta$ term in equation 21.25, will be zero. *Hence, whenever an electric dipole is placed in an external electric field, a torque will act on the dipole causing it to rotate in the external field until it becomes aligned with the external field.*

Example 21.5

Torque acting on a dipole in an external electric field. The electric dipole of example 21.4 is placed in a uniform electric field of $\mathbf{E} = (500 \text{ N/C}) \mathbf{i}$, at an angle of 35.0° with the x -axis. Find the torque acting on the dipole.

Solution

The torque acting on the dipole is found from equation 21.25 as

$$\begin{aligned} \tau &= pE \sin\theta \\ \tau &= (1.00 \times 10^{-8} \text{ m C})(500 \text{ N/C})\sin 35.0^\circ \\ \tau &= 2.87 \times 10^{-6} \text{ m N} \end{aligned}$$

[To go to this Interactive Example click on this sentence.](#)

21.6 Electric Fields of Continuous Charge Distributions

As we saw in section 21.3, when there are multiple discrete charges in a region, then the electric field produced by those charges at any point is found by the vector sum of the electric fields associated with each of the charges. That is, the resultant electric field intensity for a group of point charges was given by equation 21.10 as

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \dots \quad (21.10)$$

Equation 21.10 can be written in the shorthand notation as

$$\mathbf{E} = \sum_{i=1}^N \mathbf{E}_i \quad (21.26)$$

where, again, Σ means “the sum of” and the sum goes from $i = 1$ to $i = N$, the total number of charges present.

If the charge distribution is a continuous one, the field it sets up at any point P can be computed by dividing the continuous distribution of charge into a large number of infinitesimal elements of charge, dq . Each element of charge dq acts like a point charge and will produce an element of electric field intensity, $d\mathbf{E}$, at the point P , given by

$$d\mathbf{E} = k \frac{dq}{r^2} \mathbf{r}_o \quad (21.27)$$

where r is the distance from the element of charge dq to the field point P . \mathbf{r}_o is a unit vector that points radially away from the element of charge dq and will point toward the field point P . The total electric field intensity \mathbf{E} at the point P caused by the electric field from the entire distribution of all the dq 's is again a sum, but since the elements of charge dq are infinitesimal, the sum becomes the integral of all the elements of electric field $d\mathbf{E}$. That is, *the total electric field of a continuous distribution of charge is found as*

$$\mathbf{E} = \int d\mathbf{E} = \int k \frac{dq}{r^2} \mathbf{r}_o \quad (21.28)$$

We will now look at some specific examples of the electric fields caused by continuous charge distributions.

21.7 The Electric Field on Axis of a Charged Rod

Let us find the electric field at the point P , the origin of our coordinate system in figure 21.12, for a rod of charge that lies along the x -axis. The charge q is

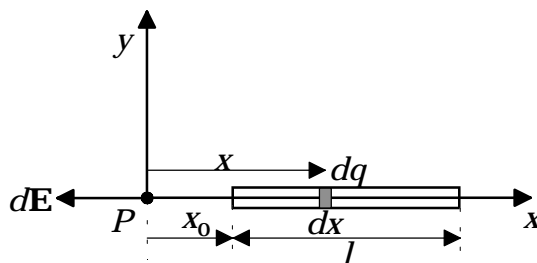


Figure 21.12 Electric field on axis for a rod of charge.

distributed uniformly over the rod. We divide the rod up into small elements of charge dq as shown. Each of these elements of charge will produce an element of electric field $d\mathbf{E}$. The element of charge dq located at the position x will produce the element of electric field $d\mathbf{E}$ given by

$$d\mathbf{E} = k \frac{dq}{x^2} (-\mathbf{i}) \quad (21.29)$$

The $-\mathbf{i}$ indicates that the element of electric field points in the negative x -direction. The total electric field at the point P is the sum or integral of each of these $d\mathbf{E}$'s and is given by equation 21.28 as

$$\mathbf{E} = \int d\mathbf{E} = \int k \frac{dq}{r^2} \mathbf{r}_o \quad (21.28)$$

$$\mathbf{E} = \int d\mathbf{E} = \int k \frac{dq}{x^2} (-\mathbf{i}) \quad (21.30)$$

The *linear charge density* λ is defined as the charge per unit length and can be written for the rod as

$$\lambda = q/x \quad (21.31)$$

The total charge on the rod can now be written as

$$q = \lambda x \quad (21.32)$$

and its differential by

$$dq = \lambda dx \quad (21.33)$$

Substituting equation 21.33 back into equation 21.30 gives

$$\mathbf{E} = - \int k \frac{dq}{x^2} \mathbf{i} = - \int k \frac{\lambda dx}{x^2} \mathbf{i}$$

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The integration is over x and as can be seen from the figure, the limits of integration will go from x_0 to $x_0 + l$, where l is the length of the rod. That is,

$$\mathbf{E} = - \int_{x_0}^{x_0+l} k\lambda \frac{dx}{x^2} \mathbf{i}$$

But k is a constant and λ , the charge per unit length, is also a constant because the charge is distributed uniformly over the rod and can be taken outside of the integral. Therefore,

$$\begin{aligned} \mathbf{E} &= -k\lambda \int_{x_0}^{x_0+l} \frac{dx}{x^2} \mathbf{i} = -k\lambda \int_{x_0}^{x_0+l} x^{-2} dx \mathbf{i} \\ \mathbf{E} &= -k\lambda [-x^{-1}]_{x_0}^{x_0+l} \mathbf{i} \\ \mathbf{E} &= -k\lambda \left[\frac{-1}{x_0+l} - \frac{-1}{x_0} \right] \mathbf{i} \\ \mathbf{E} &= -k\lambda \left[\frac{1}{x_0} - \frac{1}{x_0+l} \right] \mathbf{i} \\ \mathbf{E} &= -k\lambda \left[\frac{x_0+l-x_0}{(x_0)(x_0+l)} \right] \mathbf{i} \end{aligned}$$

Hence, the electric field at the point P is found to be

$$\mathbf{E} = - \frac{k\lambda l}{(x_0)(x_0+l)} \mathbf{i} \quad (21.34)$$

If we prefer we can write the electric field in terms of the total charge q on the rod instead of the linear charge density l , since $\lambda = q/l$ from equation 21.31. That is, the total electric field at the point P caused by the rod of charge is given by

$$\mathbf{E} = - \frac{kq}{(x_0)(x_0+l)} \mathbf{i} \quad (21.35)$$

Example 21.6

Force on a charge that is on the same axis of a rod of charge. A charge $q_0 = 5.50 \mu\text{C}$ is placed at the origin and a rod of uniform charge density of $200 \mu\text{C/m}$ is located on the x axis at $x_0 = 10.0 \text{ cm}$. The rod has a length of 12.7 cm . Find the force acting on the charge q_0 .

Solution

The total charge on the rod is found from equation 21.32 as

$$\begin{aligned} q &= \lambda x \\ q &= (200 \times 10^{-6} \text{ C/m})(0.127 \text{ m}) \\ q &= 2.54 \times 10^{-5} \text{ C} \end{aligned}$$

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The electric field at the point P is found from equation 21.35 as

$$\mathbf{E} = -\frac{kq}{(x_0)(x_0 + l)} \mathbf{i}$$

$$\mathbf{E} = -\frac{(9.00 \times 10^9 \text{ N m}^2/\text{C}^2)(2.54 \times 10^{-5} \text{ C})}{(0.10 \text{ m})(0.10 \text{ m} + 0.127 \text{ m})} \mathbf{i}$$

$$\mathbf{E} = -(1.01 \times 10^7 \text{ N/C})\mathbf{i}$$

The force acting on the charge q_0 is found from equation 21.4 as

$$\mathbf{F} = q_0\mathbf{E}$$

$$\mathbf{F} = (5.50 \times 10^{-6} \text{ C})(-1.01 \times 10^7 \text{ N/C}) \mathbf{i}$$

$$\mathbf{F} = -(55.5 \text{ N})\mathbf{i}$$

To go to this Interactive Example click on this sentence.

21.8 The Electric Field on Axis for a Ring of Charge

Let us determine the electric field at the point P , a distance x from the center of a ring of charge of radius “ a ” as shown in figure 21.13. We will assume that the

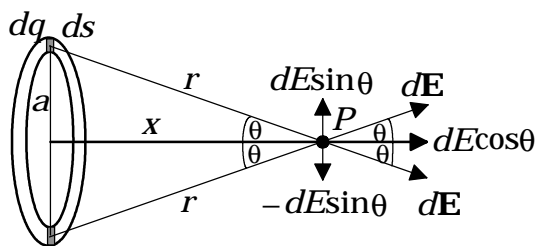


Figure 21.13 The electric field of a ring of charge.

charge is distributed uniformly along the ring, and the ring contains a total charge q . The charge per unit length of the ring, λ , is defined as

$$\lambda = q/s \quad (21.36)$$

where s is the entire length or arc of the ring (circumference). Let us now consider a small element ds at the top of the ring that contains a small element of charge dq . The total charge contained in this element dq is found from equation 21.36 as

$$q = \lambda s \quad (21.37)$$

Hence,

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$$dq = \lambda ds \quad (21.38)$$

This element of charge dq can be considered as a point charge and it sets up a differential electric field $d\mathbf{E}$ whose magnitude is given by

$$dE = k \frac{dq}{r^2} \quad (21.39)$$

and is shown in figure 21.13. The total electric field at the point P is obtained by adding up, integrating, all the small element $d\mathbf{E}$'s caused by all the dq 's. That is,

$$\mathbf{E} = \int d\mathbf{E} \quad (21.40)$$

As can be seen in the diagram, the differential of the electric field has two components, $dE \cos\theta$ and $-dE \sin\theta$. To simplify the integration we make use of the symmetry of the problem by noting that the element dq at the top of the ring will have the component $-dE \sin\theta$, while the element dq of charge at the bottom of the ring will have a component $+dE \sin\theta$. Hence, by symmetry the term $dE \sin\theta$ will always have another term that is equal and opposite, and hence the sum of all the $dE \sin\theta$'s will add to zero. Hence only the $dE \cos\theta$'s will contribute to the electric field, and thus the electric field will point along the axis of the ring in the \mathbf{i} direction. Therefore, the total electric field can now be written as

$$\mathbf{E} = \int dE \cos\theta \mathbf{i} \quad (21.41)$$

Replacing equation 21.39 into equation 21.41 we get

$$\mathbf{E} = \int dE \cos\theta \mathbf{i} = \int k \frac{dq}{r^2} \cos\theta \mathbf{i} \quad (21.42)$$

From the diagram we see that

$$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{a^2 + x^2}} \quad (21.43)$$

Replacing equations 21.38 and 21.43 into equation 21.42 we get

$$\mathbf{E} = \int k \frac{dq}{r^2} \frac{x}{\sqrt{a^2 + x^2}} \mathbf{i} = \int kx \frac{dq}{r^3} \mathbf{i} = \int k \frac{x}{(a^2 + x^2)^{3/2}} \lambda ds \mathbf{i}$$

But k , x , λ , and a are constants and can be taken outside of the integral. Thus,

$$\mathbf{E} = \frac{kx\lambda}{(a^2 + x^2)^{3/2}} \int ds \mathbf{i} \quad (21.44)$$

The integration is over ds which is an element of arc of the ring. But the sum of all the ds 's of the ring is just the circumference of the ring itself, that is,

$$\int ds = 2\pi a \quad (21.45)$$

Replacing equation 21.45 into equation 21.44 gives

$$\mathbf{E} = \frac{kx\lambda}{(a^2 + x^2)^{3/2}}(2\pi a) \mathbf{i} \quad (21.46)$$

But the charge per unit length, $\lambda = q/s = q/(2\pi a)$ from equation 21.36, hence substituting this back into equation 21.46 we get

$$\mathbf{E} = \frac{kx}{(a^2 + x^2)^{3/2}} \frac{q}{(2\pi a)}(2\pi a) \mathbf{i} \quad (21.47)$$

Therefore the electric field at the point x due to a ring of charge of radius “ a ” carrying a total charge q is given by

$$\mathbf{E} = \frac{kqx}{(a^2 + x^2)^{3/2}} \mathbf{i} \quad (21.48)$$

Example 21.7

Electric field on axis of a ring of charge at a very large distance from the ring. Find the electric field on the axis of a ring of charge at a very large distance from the ring of charge. That is, assume $x \gg a$.

Solution

If $x \gg a$ then as an approximation

$$(a^2 + x^2)^{3/2} \approx x^3$$

Replacing this assumption into equation 21.48 yields

$$\begin{aligned} \mathbf{E} &= \frac{kqx}{(a^2 + x^2)^{3/2}} \mathbf{i} = \frac{kqx}{x^3} \mathbf{i} \\ \mathbf{E} &= \frac{kq}{x^2} \mathbf{i} \end{aligned}$$

But this is the electric field of a point charge. Thus at very large distances from the ring of charge, the ring of charge looks like a point charge as would be expected.

Example 21.8

Electric field at the center of a ring of charge. Find the electric field at the center of a ring of charge.

Solution

At the center of the ring $x = 0$. Therefore the electric field at the center of a ring of charge is found from equation 21.48 with x set equal to zero. That is,

$$\mathbf{E} = \frac{kqx}{(a^2 + x^2)^{3/2}} \mathbf{i} = \frac{kq(0)}{(a^2 + 0)^{3/2}} \mathbf{i}$$

$$\mathbf{E} = 0$$

Example 21.9

Maximum value of the electric field of a ring of charge. Find the point on the x -axis where the electric field of a ring of charge takes on its maximum value.

Solution

As you recall from your calculus course the maximum value of a function is found by taking the first derivative of the function with respect to x and setting that derivative equal to zero. Hence, the maximum value of the electric field is found by taking the first derivative of equation 21.48 and setting it equal to zero. Therefore,

$$\frac{dE}{dx} = \frac{d}{dx} \left[\frac{kqx}{(a^2 + x^2)^{3/2}} \right] = \frac{d}{dx} [kqx(a^2 + x^2)^{-3/2}] = 0$$

$$\frac{dE}{dx} = kq [x(-3/2)(a^2 + x^2)^{-5/2}(2x) + (a^2 + x^2)^{-3/2}] = 0$$

$$2x^2(-3/2)(a^2 + x^2)^{-5/2} + (a^2 + x^2)^{-3/2} = 0$$

$$x^2(-3)(a^2 + x^2)^{-5/2} = -(a^2 + x^2)^{-3/2}$$

$$3x^2(a^2 + x^2)^{-2/2} = 1$$

$$3x^2 = (a^2 + x^2)$$

$$3x^2 - x^2 = a^2$$

$$2x^2 = a^2$$

$$x = \frac{\pm a}{\sqrt{2}}$$

Thus, the maximum value of the electric field on the x -axis occurs at $x = \pm a/\sqrt{2}$, or approximately at 7/10 of the radius of the ring in front of or behind the ring.

21.9 The Electric Field on Axis for a Disk of Charge

Let us find the electric field on axis at the point P in figure 21.14(a) for a uniform disk of charge. Since a disk can be generated by adding up many rings of

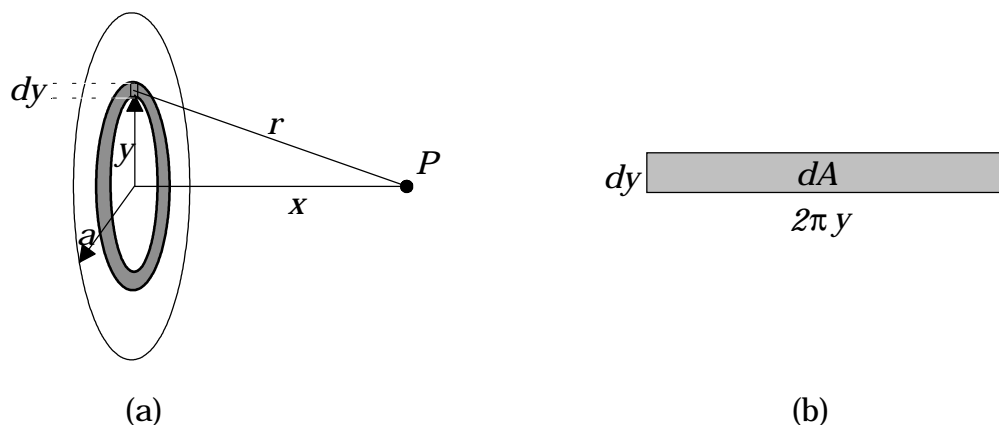


Figure 21.14 The electric field of a disk of charge.

different radii, the electric field of a disk of charge can be generated by adding up (integrating) the electric field of many rings of charge. Thus the electric field of a disk of charge will be given by

$$\mathbf{E}_{\text{disk}} = \int d\mathbf{E}_{\text{ring}} \quad (21.49)$$

We found in the last section that the electric field on axis at the distance x from the center of a ring of charge of radius “ a ” was given by

$$\mathbf{E} = \frac{kqx}{(a^2 + x^2)^{3/2}} \mathbf{i} \quad (21.48)$$

In the present problem the ring will be of radius y and we will add up all the rings from a radius of 0 to the radius “ a ”, the radius of the disk. Hence, equation 21.48 will be written as

$$\mathbf{E}_{\text{ring}} = \frac{kqx}{(y^2 + x^2)^{3/2}} \mathbf{i} \quad (21.50)$$

We now consider the charge on this ring to be a small element dq of the total charge that will be found on the disk. This element of charge dq will then produce an element of electric field $d\mathbf{E}$, that lies along the axis of the disk in the \mathbf{i} direction. That is,

$$d\mathbf{E}_{\text{ring}} = \frac{kx}{(y^2 + x^2)^{3/2}} dq \mathbf{i} \quad (21.51)$$

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Replacing equation 21.51 back into equation 21.49 for the electric field of the disk we get

$$\mathbf{E}_{\text{disk}} = \int d\mathbf{E}_{\text{ring}} = \int \frac{kx}{(y^2 + x^2)^{3/2}} dq \mathbf{i} \quad (21.52)$$

When dealing with a rod of charge or a ring of charge which has the charge distributed along a line, we introduced the concept of the *linear charge density* λ as *the charge per unit length*. When dealing with a disk, the electric charge is distributed across a surface. Therefore, *we now define a surface charge density* σ as *the charge per unit area* and it is given by

$$\sigma = \frac{q}{A} \quad (21.53)$$

In terms of the surface charge density, the charge on the disk is given by

$$q = \sigma A \quad (21.54)$$

Its differential dq , is the amount of charge on the ring, i.e.,

$$dq = \sigma dA \quad (21.55)$$

where dA is the area of the ring. To determine the area of the ring, let us take the ring in figure 21.14(a) and unfold it as shown in figure 21.14(b). The length of the ring is the circumference of the inner circle of the ring, $2\pi y$, and its width is the differential thickness of the ring, dy . The area of the ring dA is then given by the product of its length and width as

$$dA = (2\pi y) dy \quad (21.56)$$

Replacing equation 21.56 back into 21.55 gives for the element of charge of the ring

$$dq = \sigma(2\pi y) dy \quad (21.57)$$

Replacing equation 21.57 back into equation 21.52 we get for the electric field of the disk

$$\mathbf{E}_{\text{disk}} = \int \frac{kx}{(y^2 + x^2)^{3/2}} \sigma 2\pi y dy \mathbf{i} \quad (21.58)$$

Taking the constants outside of the integral we get

$$\begin{aligned} \mathbf{E}_{\text{disk}} &= kx2\pi\sigma \int \frac{y dy}{(y^2 + x^2)^{3/2}} \mathbf{i} \\ \mathbf{E}_{\text{disk}} &= \left(\frac{1}{4\pi\epsilon_0} \right) x2\pi\sigma \int \frac{y dy}{(y^2 + x^2)^{3/2}} \mathbf{i} \end{aligned}$$

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$$\mathbf{E}_{\text{disk}} = \left(\frac{\sigma X}{2\epsilon_0} \right) \int_0^a \frac{y dy}{(y^2 + x^2)^{3/2}} \mathbf{i} \quad (21.60)$$

Note that we have introduced the limits of integration 0 to a , that is, we add up all the rings from a radius of 0 to the radius “ a ”, the radius of the disk. To determine the electric field it is necessary to solve the integral

$$I = \int \frac{y dy}{(y^2 + x^2)^{3/2}}$$

Let us make the substitution

$$u = (y^2 + x^2) \quad (21.61)$$

Hence, its differential is

$$du = 2y dy$$

Also

$$(y^2 + x^2)^{3/2} = u^{3/2}$$

and

$$y dy = du/2$$

Replacing these substitutions into our integral we obtain

$$\begin{aligned} I &= \int \frac{y dy}{(y^2 + x^2)^{3/2}} = \int \frac{\frac{du}{2}}{u^{3/2}} \\ I &= \int \frac{1}{2} u^{-3/2} du \\ I &= \frac{1}{2} \frac{u^{(-\frac{3}{2})+1}}{(-\frac{3}{2}+1)} = -u^{-1/2} = \frac{-1}{u^{1/2}} \end{aligned}$$

Hence

$$\begin{aligned} I &= \int_0^a \frac{y dy}{(y^2 + x^2)^{3/2}} = -\frac{1}{(y^2 + x^2)^{1/2}} \Big|_0^a \\ I &= \int_0^a \frac{y dy}{(y^2 + x^2)^{3/2}} = -\frac{1}{(a^2 + x^2)^{1/2}} - \frac{-1}{(0 + x^2)^{1/2}} \\ I &= \int_0^a \frac{y dy}{(y^2 + x^2)^{3/2}} = \frac{1}{x} - \frac{1}{(a^2 + x^2)^{1/2}} \end{aligned} \quad (21.62)$$

Replacing equation 21.62 back into equation 21.60 we obtain

$$\begin{aligned} \mathbf{E}_{\text{disk}} &= \left(\frac{\sigma X}{2\epsilon_0} \right) \int_0^a \frac{y dy}{(y^2 + x^2)^{3/2}} \mathbf{i} = \left(\frac{\sigma X}{2\epsilon_0} \right) \left[\frac{1}{x} - \frac{1}{(a^2 + x^2)^{1/2}} \right] \mathbf{i} \\ \mathbf{E}_{\text{disk}} &= \left(\frac{\sigma}{2\epsilon_0} \right) \left[1 - \frac{x}{(a^2 + x^2)^{1/2}} \right] \mathbf{i} \end{aligned} \quad (21.63)$$

Equation 21.63 gives the electric field at the position x on the axis of a disk of charge of radius “ a ”, carrying a uniform surface charge density σ . The direction of

the electric field of the disk of charge is in the \mathbf{i} -direction just as the electric field of the ring of charge.

Example 21.10

Electric field on axis of a charged disk. (a) Find the electric field at the point $x = 15.0$ cm in front of a disk of 10.0 cm radius carrying a uniform surface charge density of $200 \mu\text{C}/\text{m}^2$. (b) Find the force acting on an electron placed at this point.

Solution

a. The electric field of the disk of charge is found from equation 21.63 as

$$\mathbf{E}_{\text{disk}} = \left(\frac{\sigma}{2\epsilon_0} \right) \left[1 - \frac{x}{(a^2 + x^2)^{1/2}} \right] \mathbf{i} \quad (21.63)$$

$$\mathbf{E}_{\text{disk}} = \left(\frac{200 \times 10^{-6} \text{ C/m}^2}{2(8.85 \times 10^{-12} \text{ C}^2/\text{N m}^2)} \right) \left[1 - \frac{0.150 \text{ m}}{((0.100 \text{ m})^2 + (0.150)^2)^{1/2}} \right] \mathbf{i}$$

$$\mathbf{E}_{\text{disk}} = (1.13 \times 10^7 \text{ N/C}) [1 - 0.832] \mathbf{i}$$

$$\mathbf{E}_{\text{disk}} = (1.90 \times 10^6 \text{ N/C}) \mathbf{i}$$

b. The force on an electron placed at this position is found as

$$\mathbf{F} = q\mathbf{E}$$

$$\mathbf{F} = (-1.60 \times 10^{-19} \text{ C})(1.90 \times 10^6 \text{ N/C}) \mathbf{i}$$

$$\mathbf{F} = -(3.04 \times 10^{-13} \text{ N}) \mathbf{i}$$

To go to this Interactive Example click on this sentence.

21.10 Dynamics of a Charged Particle in an Electric Field

When an electric charge q is placed in an electric field, it experiences a force given by

$$\mathbf{F} = q\mathbf{E} \quad (21.4)$$

If the electric charge is free to move, then it will experience an acceleration given by Newton's second law as

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{q\mathbf{E}}{m} \quad (21.64)$$

If the electric field is a constant field, then the acceleration is a constant and the kinematic equations developed in chapter 4 can be used to describe the motion of the charged particle. The position of the particle at any instant of time can be found by

$$\mathbf{r} = \mathbf{v}_0 t + \frac{1}{2} \mathbf{a} t^2 \quad (4.5)$$

and its velocity by

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a} t \quad (4.6)$$

where the acceleration \mathbf{a} is given by equation 21.64.

Example 21.11

Projectile motion of a charged particle in an electric field. A proton is fired at an initial velocity of 150 m/s at an angle of 60.0° above the horizontal into a uniform electric field of 2.00×10^{-4} N/C between two charged parallel plates, as shown in figure 21.15. Find (a) the total time the particle is in motion, (b) its maximum range, and (c) its maximum height. Neglect any gravitational effects.

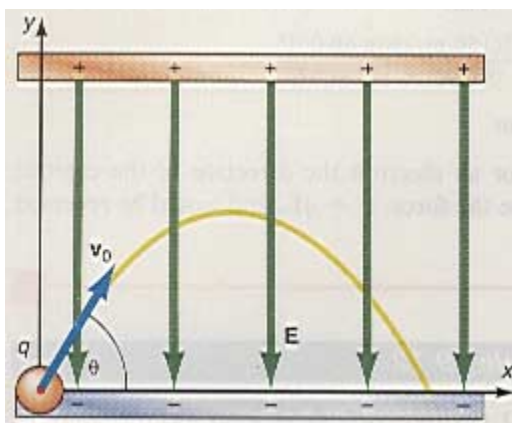


Figure 21.15 Dynamics of a charged particle in an external electric field.

Solution

The acceleration of the proton, determined from equation 21.64, is

$$\begin{aligned} a &= \frac{qE}{m} = \frac{(1.60 \times 10^{-19} \text{ C})(2.00 \times 10^{-4} \text{ N/C})}{1.67 \times 10^{-27} \text{ kg}} \\ &= 1.92 \times 10^4 \text{ m/s}^2 \end{aligned}$$

The acceleration is in the direction of \mathbf{E} , which is in the negative y -direction. There is no acceleration in the x -direction because the component of \mathbf{E} in the x -direction is zero. The kinematic equations in component form are

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$$\begin{aligned}
 y &= v_{0y}t - \frac{1}{2}at^2 \\
 x &= v_{0x}t \\
 v_y &= v_{0y} - at \\
 v_x &= v_{0x} \\
 v_y^2 &= v_{0y}^2 - 2ay
 \end{aligned}$$

Notice that they have the same form as they did in the discussion of the problem of projectile motion on the surface of the earth.

a. To determine the total time that the particle is in flight, we make use of the fact that when the time t is equal to the total time that the particle is in flight, $t = t_t$, the projectile has returned to the bottom plate and its height is equal to zero, that is, $y = 0$. Thus,

$$\begin{aligned}
 0 &= v_{0y}t_t - \frac{1}{2}at_t^2 \\
 t_t &= \frac{v_{0y}}{a} = \frac{2v_o \sin \theta}{a} \\
 &= \frac{2(150 \text{ m/s})\sin 60.0^\circ}{1.92 \times 10^4 \text{ m/s}^2} \\
 &= 1.35 \times 10^{-2} \text{ s}
 \end{aligned}$$

b. The range of the particle is the maximum distance that the projectile moves in the x -direction, hence

$$\begin{aligned}
 x_{\max} &= v_{0x}t_t = (v_o \cos \theta)t_t \\
 &= [(150 \text{ m/s})\cos 60.0^\circ](1.35 \times 10^{-2} \text{ s}) \\
 &= 1.01 \text{ m}
 \end{aligned}$$

c. The maximum height of the projectile is determined by using the fact that when $y = y_{\max}$, the projectile is at the top of the trajectory and $v_y = 0$. Hence,

$$\begin{aligned}
 v_y^2 &= v_{0y}^2 - 2ay \\
 0 &= v_{0y}^2 - 2ay_{\max} \\
 y_{\max} &= \frac{v_{0y}^2}{2a} \\
 &= \frac{[(150 \text{ m/s})\sin 60.0^\circ]^2}{2(1.92 \times 10^4 \text{ m/s}^2)} \\
 &= 0.439 \text{ m}
 \end{aligned}$$

To obtain the same projectile motion for an electron the direction of the electric field \mathbf{E} would have to be reversed because the force, $\mathbf{F} = q\mathbf{E}$, and would be reversed when q is negative.

To go to this Interactive Example click on this sentence.

Example 21.12

Motion of an electron in a cathode ray oscilloscope. A cathode ray oscilloscope works by deflecting a beam of electrons as they pass through a uniform electric field between two parallel plates, as shown in figure 21.16. The deflected beam falls on a fluorescent screen, where it is visible as a dot. If the plates are 2.00 cm long, the initial velocity of the electron is $v_0 = 3.00 \times 10^7$ m/s, and the electric field between the plates is $E = 2.00 \times 10^4$ N/C, (a) what is the y -position of the electron as it leaves the field? (b) If the distance from the end of the plates to the screen is 60.0 cm, find the actual y -position on the screen.

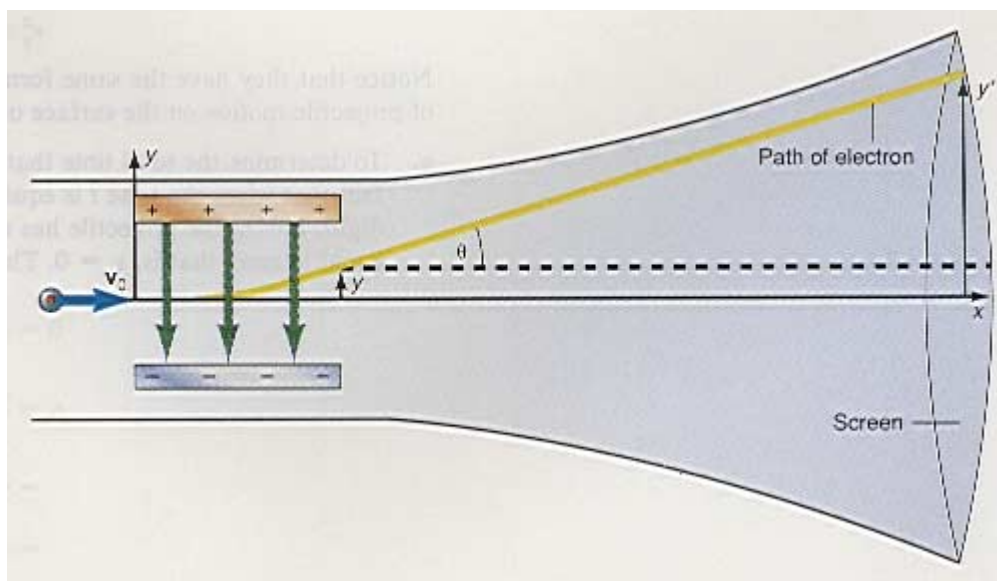


Figure 21.16 The dynamics of an electron in an oscilloscope.

Solution

Note that the electron is deflected opposite to the direction of the field. This is because the electron is negative. The time it takes for the electron to traverse the field is found from the kinematic equation

$$\begin{aligned}
 x &= v_{0x}t \\
 t &= \frac{x}{v_{0x}} \\
 &= \frac{0.0200 \text{ m}}{3.00 \times 10^7 \text{ m/s}} \\
 &= 6.67 \times 10^{-10} \text{ s}
 \end{aligned}$$

The acceleration of the electron in the field, found from equation 21.33, is

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$$\begin{aligned}
 a &= \frac{qE}{m} \\
 &= \frac{(1.60 \times 10^{-19} \text{ C})(2.00 \times 10^4 \text{ N/C})}{9.11 \times 10^{-31} \text{ kg}} \\
 &= 3.51 \times 10^{15} \text{ m/s}^2
 \end{aligned}$$

Therefore, the y -position of the electron as it leaves the field is

$$\begin{aligned}
 y &= v_{0y}t + \frac{1}{2} at^2 \\
 &= 0 + \frac{1}{2} (3.51 \times 10^{15} \text{ m/s}^2)(6.67 \times 10^{-10} \text{ s})^2 \\
 &= 7.81 \times 10^{-4} \text{ m} \\
 &= 0.781 \text{ mm}
 \end{aligned}$$

b. As the electron leaves the field it travels in a straight line making an angle θ with the x -axis, as shown in figure 21.17. At the moment it leaves, its x -component

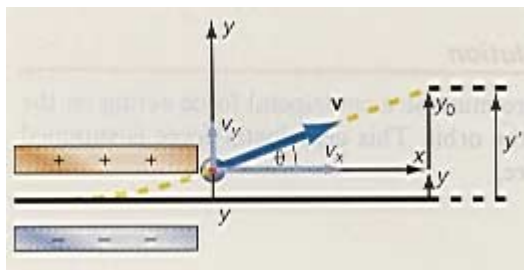


Figure 21.17 Location of electron as it hits the screen.

of velocity is $v_x = v_{0x} = v_0$, whereas its y -component of velocity is found from

$$\begin{aligned}
 v_y &= v_{0y} + at = 0 + (3.51 \times 10^{15} \text{ m/s}^2)(6.67 \times 10^{-10} \text{ s}) \\
 &= 2.34 \times 10^6 \text{ m/s}
 \end{aligned}$$

The angle θ at which the electron leaves the field is found from

$$\begin{aligned}
 \theta &= \tan^{-1} \frac{v_y}{v_x} \\
 &= \tan^{-1} \frac{2.34 \times 10^6 \text{ m/s}}{3.00 \times 10^7 \text{ m/s}} \\
 &= 4.46^\circ
 \end{aligned}$$

If the distance from the end of the plates to the screen is 60.0 cm, then the deflection on the screen, found from figure 21.17, is

$$y_0 = x \tan \theta$$

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$$\begin{aligned} &= (60.0 \text{ cm})\tan 4.46^\circ \\ &= 4.68 \text{ cm} \end{aligned}$$

Adding this to y gives the actual deflection y' from the center of the screen as

$$y' = y_0 + y = 4.68 \text{ cm} + 0.078 \text{ cm} = 4.76 \text{ cm}$$

By varying the value of the electric field between the plates any deflection can be obtained in the y -direction on the screen. When the electric field is reversed in sign, the deflection is in the negative y -direction on the screen. If another pair of plates is placed vertically around the first set of plates, as shown in figure 21.18, and an electric field is also set up between them, the electron can be deflected anywhere on the x -axis. The combination of both plates can move the dot anywhere on the screen.

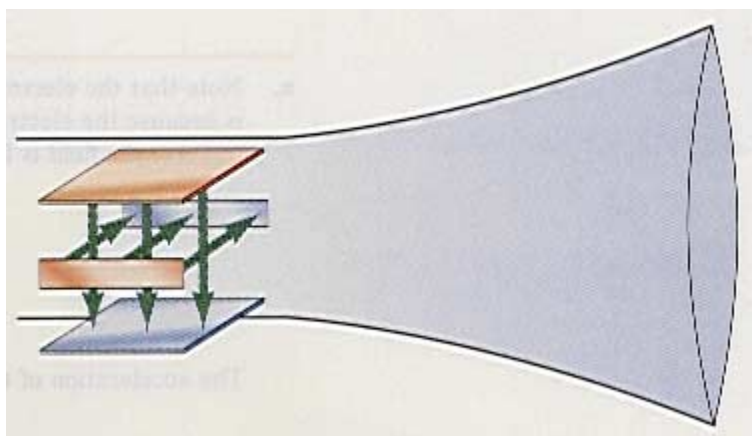


Figure 21.18 An oscilloscope with two sets of plates.

To go to this Interactive Example click on this sentence.

Example 21.13

The electron in the hydrogen atom. In the Bohr model of the hydrogen atom, an electron orbits the proton in a circular orbit. If the radius of the orbit is $r = 0.529 \times 10^{-10} \text{ m}$, what is its velocity?

Solution

If the electron is in a circular orbit, there must be a centripetal force acting on the electron directed toward the center of the orbit. This centripetal force is supplied by the Coulomb electric force. Therefore,

$$F_c = F_e$$

$$\begin{aligned}
 \frac{m_e v^2}{r} &= \frac{k q_1 q_2}{r^2} \\
 v &= \sqrt{\frac{k q^2}{m_e r^2}} \\
 &= \sqrt{\frac{(9.00 \times 10^9 \text{ N m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(9.11 \times 10^{-31} \text{ kg})(0.529 \times 10^{-10} \text{ m})}} \\
 &= 2.19 \times 10^6 \text{ m/s}
 \end{aligned}
 \tag{21.65}$$

To go to this Interactive Example click on this sentence.

The Language of Physics

Electric field

It is an intrinsic property of nature that an electric field exists in the space around an electric charge. The electric field is considered to be a force field that exerts a force on charges placed in the field (p.).

Electric field intensity

The electric field is measured in terms of the electric field intensity. The magnitude of the electric field intensity is defined as the ratio of the force acting on a small test charge to the magnitude of the small test charge. The direction of the electric field is in the direction of the force on the positive test charge (p.).

Electric field intensity of a point charge

The electric field of a point charge is directly proportional to the charge that creates it, and inversely proportional to the square of the distance from the point charge to the position where the field is being evaluated (p.).

Superposition of electric fields

When more than one charge contributes to the electric field, the resultant electric field is the vector sum of the electric fields produced by each of the various charges (p.).

Summary of Important Equations

Definition of the electric field intensity $\mathbf{E} = \frac{\mathbf{F}}{q_0}$ (21.1)

Electric field intensity of a point charge $E = \frac{kq}{r^2}$ (21.2)

$\mathbf{E} = k \frac{q}{r^2} \mathbf{r}_o$ (21.3)

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Force on a charge q in an electric field $\mathbf{F} = q\mathbf{E}$ (21.3)

Superposition principle

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \dots \quad (21.4)$$

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \dots \quad (21.9)$$

$$\mathbf{E} = \sum_{i=1}^N \mathbf{E}_i \quad (21.26)$$

Electric dipole moment $p = 2aq$ (21.15)

$$\mathbf{p} = 2aq\mathbf{r}_o \quad (21.18)$$

Electric field of a dipole along its perpendicular bisector

$$\mathbf{E} = \frac{kp}{(a^2 + x^2)^{3/2}} \mathbf{j} \quad (21.16)$$

$$\mathbf{E} = \frac{kp}{x^3} \mathbf{j} \quad (21.17)$$

Torque on a dipole in an external electric field

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} \quad (21.24)$$

$$\tau = pE \sin\theta \quad (21.25)$$

Electric field of a continuous distribution of charge $\mathbf{E} = \int d\mathbf{E} = \int k \frac{dq}{r^2} \mathbf{r}_o$ (21.28)

Electric field caused by a rod of charge $\mathbf{E} = -\frac{kq}{(x_o)(x_o + l)} \mathbf{i}$ (21.35)

Electric field due to a ring of charge $\mathbf{E} = \frac{kqx}{(a^2 + x^2)^{3/2}} \mathbf{i}$ (21.48)

Electric field of a disk of charge $\mathbf{E}_{\text{disk}} = \int d\mathbf{E}_{\text{ring}}$ (21.49)

$$\mathbf{E}_{\text{disk}} = \left(\frac{\sigma}{2\epsilon_o} \right) \left[1 - \frac{x}{(a^2 + x^2)^{1/2}} \right] \mathbf{i} \quad (21.63)$$

Acceleration of a charged particle in an electric field $\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{q\mathbf{E}}{m}$ (21.33)

Questions for Chapter 21

1. Describe as many different types of fields as you can.
- *2. Because you cannot really see an electric field, is anything gained by using the concept of a field rather than an “action at a distance” concept?
- *3. Is there any experimental evidence that can substantiate the existence of an electric field rather than the concept of an “action at a distance”?

Chapter 21 Electric Fields

4. Is the force of gravity also an “action at a distance”? Should a gravitational field be introduced to explain gravity? What is the equivalent gravitational “charge”?

*5. If there are positive and negative electrical charges, could there be positive and negative masses? If there were, what would their characteristics be?

*6. Michael Faraday introduced the concept of lines of force to explain electrical interactions. What is a line of force and how is it like an electric field line? Is there any difference?

Problems for Chapter 21

Section 21.2 The Electric Field of a Point Charge.

1. Find the electric field 2.00 m from a point charge of 3.00 pC.
2. A point charge, $q = 3.75 \mu\text{C}$, is placed in an electric field of 250 N/C. Find the force on the charge.

Section 21.3 Superposition of Electric Fields For Multiple Discrete Charges.

3. Find the electric field at point A in the diagram if (a) $q_1 = 2.00 \mu\text{C}$, and $q_2 = 3.00 \mu\text{C}$. and (b) $q_1 = 2.00 \mu\text{C}$ and $q_2 = -3.00 \mu\text{C}$.

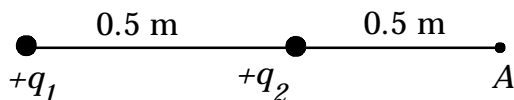


Diagram for problem 3.

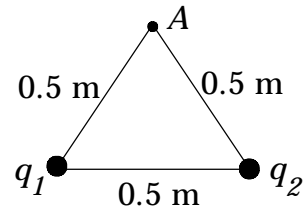


Diagram for Problem 5 and 6.

*4. A point charge of $+2.00 \mu\text{C}$ is 30.0 cm from a charge of $+3.00 \mu\text{C}$. Where is the electric field between the charges equal to zero?

*5. Find the electric field at the apex of the triangle shown in the diagram if $q_1 = 2.00 \mu\text{C}$ and $q_2 = 3.00 \mu\text{C}$. What force would act on a $6.00 \mu\text{C}$ charge placed at this point?

*6. Find the electric field at point A in the diagram if $q_1 = 2.00 \mu\text{C}$ and $q_2 = -3.00 \mu\text{C}$.

*7. Charges of $2.00 \mu\text{C}$, $4.00 \mu\text{C}$, $-6.00 \mu\text{C}$ and $8.00 \mu\text{C}$ are placed at the corner of a square of 50.0 cm length. Find the electric field at the center of the square.

Section 21.4 Electric Field of an Electric Dipole.

8. Find the electric dipole moment of a charge of $4.50 \mu\text{C}$ separated by 5.00 cm from a charge of $-4.50 \mu\text{C}$.

*9. If a charge of $2.00 \mu\text{C}$ is separated by 4.00 cm from a charge of $-2.00 \mu\text{C}$ find the electric field at a distance of 5.00 m , perpendicular to the axis of the dipole.

*10. Find the electric field at point A in the diagram if charge $q_1 = 2.63 \mu\text{C}$ and $q_2 = -2.63 \mu\text{C}$, $d = 10.0 \text{ cm}$, $r_1 = 50.0 \text{ cm}$, $\theta_1 = 25.0^\circ$. Hint: first find r_2 by the law of cosines, then with r_2 known, use the law of cosines again to find the angle θ_2 .

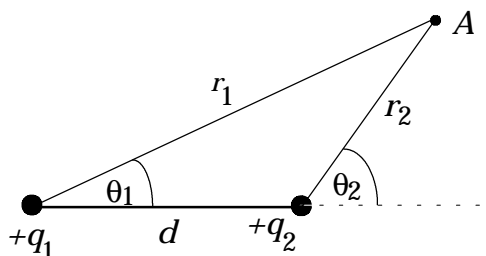


Diagram for problem 10.

Additional Problems

11. A point charge of 3.00 pC is located at the origin of a coordinate system. (a) What is the electric field at $x = 50.0 \text{ cm}$? (b) What force would act on a $2.00 \mu\text{C}$ charge placed at $x = 50.0 \text{ cm}$?

12. A point charge of $2.00 \mu\text{C}$ is 30.0 cm from a charge of $3.00 \mu\text{C}$. Find the electric field half way between the charges.

13. Starting with equation 21.63 for the electric field on the x -axis for a uniform disk of charge, find the electric field on the x -axis for an infinite sheet of charge in the y - z plane. (Hint: An infinite sheet of charge can be generated from a finite disk of charge by letting the radius of the disk of charge approach infinity.)

14. An electron is placed between two charged parallel plates. What must the value of the electric field be in order that the electron be in equilibrium between the electric force and the gravitational force?

15. Find the equation for the electric field at the point P in figure 21.12 for a rod of charge that has a nonuniform linear charge density λ given by $\lambda = Ax^2$.

16. Find the equation for the electric field at the point P in the diagram for a ring of charge that has a *nonuniform* linear charge density λ given by $\lambda = A \sin\phi$, where ϕ is the angle between the y -axis and the location of the element of charge dq , and A is a constant.

17. Starting with the equation for the electric field of a uniform ring of charge, find the electric field on the x -axis for a *nonuniform* disk of charge. The surface charge density on the rings vary linearly with the radius of the ring, i.e., $\sigma = Cy$ where y is the radius of each ring and C is a constant.

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18. Find the equation for the electric field between two disks of charge. The first one carries the charge density $+\sigma$ while the second carries the charge density $-\sigma$.

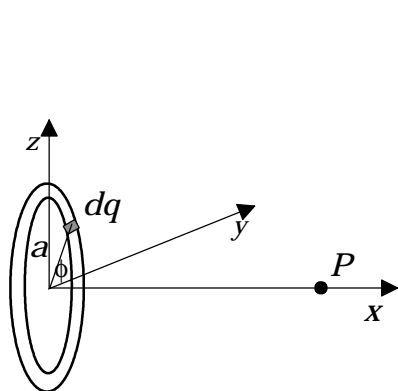


Diagram for problem 16.

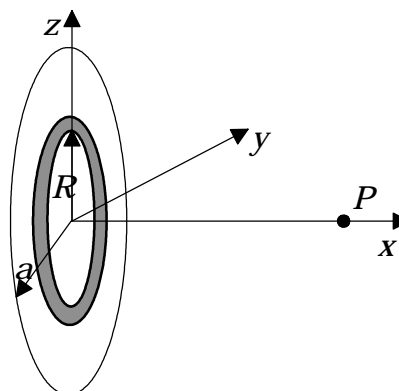


Diagram for problem 17.

19. A wire of length l , lying along the x -axis, carries a charge of λ per unit length. Find the electric field at the arbitrary point P a distance R from the wire. Show that this reduces to the solution

$$\mathbf{E} = \frac{kq}{y\sqrt{(l/2)^2 + y^2}} \mathbf{j}$$

when the point P is along the perpendicular bisector of the wire.

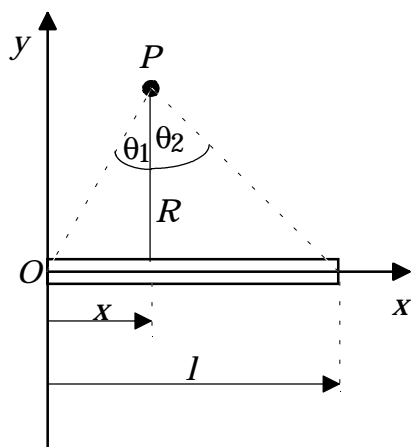


Diagram for problem 19.

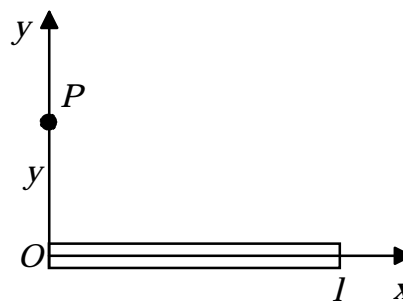


Diagram for problem 20.

20. A uniform rod of charge lies along the x -axis and carries a charge of λ per unit length. One end of the rod is located at the origin of the coordinate system, while the other end is at a distance l along the x -axis. Find the electric field at the point P a distance y along the y -axis.

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21. One line of charge lies on the x -axis while a second line of charge lies on the y -axis as shown. Find the electric field at the point P in the diagram.

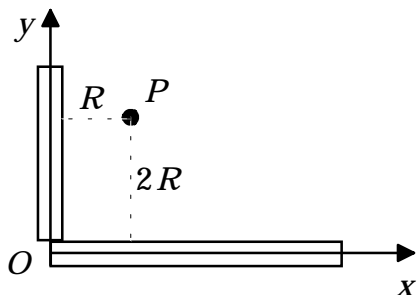


Diagram for problem 21.

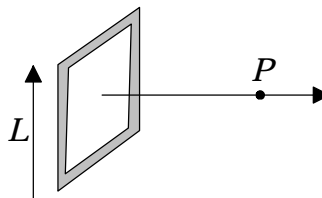


Diagram for problem 22.

22. A wire carrying a charge density λ is bent into the form of a square of length L . Find the electric field at the point P perpendicular to the plane of the square, on a line passing through the center of the square.

23. Show that the rectangular components of the electric field of a dipole for an arbitrary point P in the x - y plane are given by

$$E_x = \frac{k3pxy}{(x^2 + y^2)^{5/2}}$$

$$E_y = \frac{kp(2y^2 - x^2)}{(x^2 + y^2)^{5/2}}$$

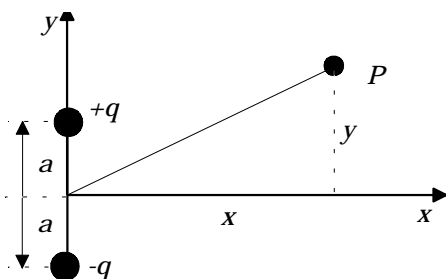


Diagram for problem 23.

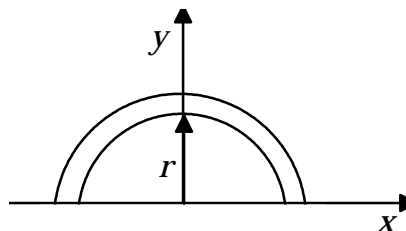


Diagram for problem 24.

24. A thin rod carrying a charge q spread uniformly along its length is bent into a semicircle of radius r . Find the electric field at the center of the semicircle.

25. A thin rod carrying a charge q spread uniformly along its length is bent into an arc of a circle of radius r . The arc subtends an angle θ_0 , as shown in the diagram. Find the electric field at the center of the circle.

26. A hemispherical insulating cup of inner radius R carries a total charge q over its surface. Find the electric field at the center of the cup.

27. Find the point on the x -axis where the electric field of a disk of charge takes on its maximum value.

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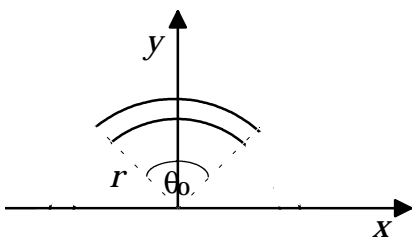


Diagram for problem 25.

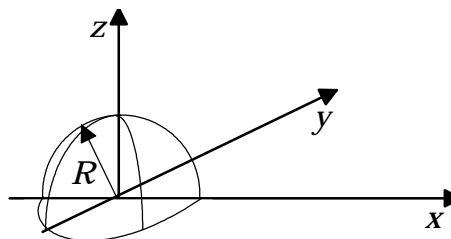


Diagram for problem 26.

28. A sphere of 10.0 cm radius has a charge of 8.00 μC distributed uniformly over its surface. Find the surface charge density σ .

29. A sphere of 10.0 cm radius has a charge of 8.00 μC distributed uniformly throughout its volume. Find the volume charge density ρ .

30. A sphere has a surface charge density σ given by $\sigma = \sigma_0 r$. Find the total charge contained on the surface of the sphere.

31. A sphere has a volume charge density ρ given by $\rho = \rho_0/r^2$. Find the total charge contained within the sphere.

32. A 10.0 g pith ball, carrying a charge of 1.52 μC hangs from a 25.0 cm thread attached to a vertical wall of charge carrying a surface charge density $\sigma = 3.57 \mu\text{C}/\text{m}^2$. Find the angle that the thread makes with the vertical.

33. Find the electric field at the point P halfway between an infinite sheet of charge carrying a surface charge density $\sigma = 84.5 \mu\text{C}/\text{m}^2$ and a point charge carrying a charge $q = 5.25 \mu\text{C}$ as shown in the diagram. The distance d between the sheet of charge and the point charge is 7.55 cm.

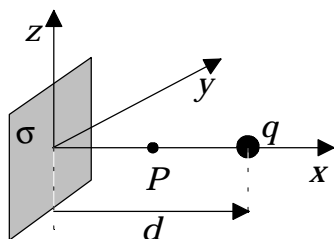


Diagram for problem 33.

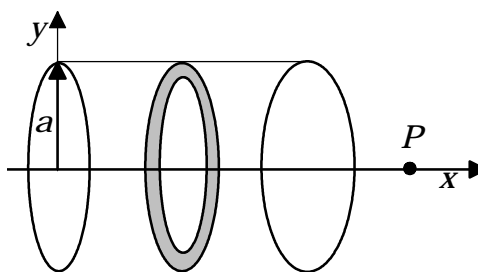


Diagram for problem 34.

34. Find the electric field \mathbf{E} on the x -axis at the point P of the cylindrical shell of radius a shown in the diagram. Hint: the cylindrical shell is made up of a sum of rings of charges.

35. Find the electric field \mathbf{E} on the x -axis at the point P of the solid cylinder of radius a shown in the diagram. Hint: the solid cylinder is made up of a sum of disks of charges.

36. Three infinite sheets of charge are arranged as shown. Sheet 1 carries a surface charge density $\sigma_1 = 1 \mu\text{C}/\text{m}^2$, sheet 2 carries a surface charge density $\sigma_2 = 2$

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$\mu\text{C}/\text{m}^2$, while sheet 3 carries a surface charge density $\sigma_3 = 3 \mu\text{C}/\text{m}^2$. Find the electric field \mathbf{E} in regions 1, 2, 3, and 4.

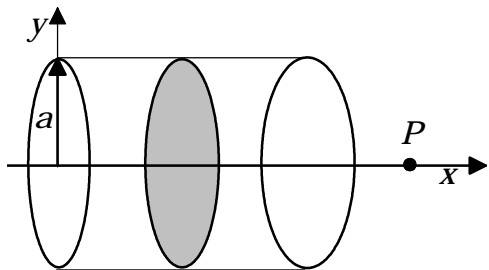


Diagram for problem 35.

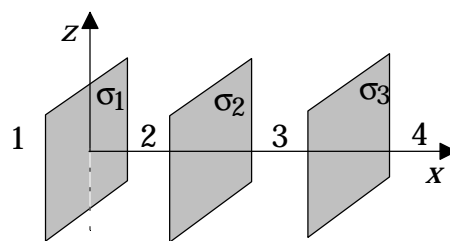


Diagram for problem 36.

37. Show that the solution for the electric field on axis of a ring of charge, equation 21.48, reduces to the electric field of a point charge if x is very much greater than a , the radius of the ring. Hint: use the binomial theorem.

38. Two charged rods, one carrying a positive charge density λ and one carrying a negative charge density $-\lambda$ are bent into the semicircle shown. Find the equation for the electric field \mathbf{E} at the point P .

39. Find the electric field at the point P midway between a disk of charge carrying a surface charge density $\sigma = 250 \mu\text{C}/\text{m}^2$ and a ring of charge carrying a total charge $q = 5.60 \mu\text{C}$. The radius of the disk is $a_{\text{disk}} = 0.150 \text{ m}$, and the radius of the ring is $a_{\text{ring}} = 0.150 \text{ m}$.

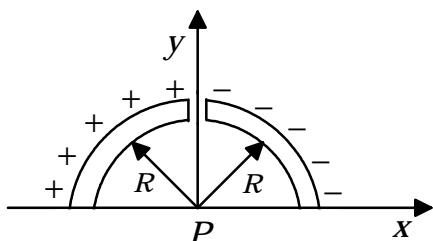


Diagram for problem 38.

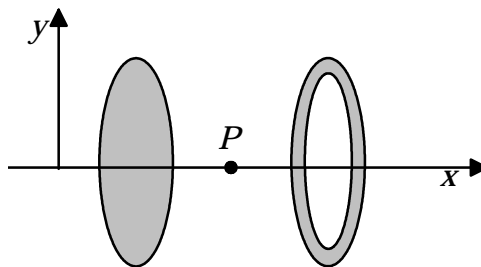


Diagram for problem 39.

40. Find the electric field \mathbf{E} at the origin of the coordinate system shown for the two rods of charge. The linear charge density $\lambda_1 = 20.0 \mu\text{C}/\text{m}$ while $\lambda_2 = 35.0 \mu\text{C}/\text{m}$. The distance $a = 0.150 \text{ m}$ while the distance $b = 0.250 \text{ m}$.

41. Two essentially infinite lines of charge in the y - z plane are carrying linear charge densities of $+\lambda$ and $-\lambda$. Find the equation for the electric field at the arbitrary point P . What would the electric dipole moment be?

42. The parallel plates of a cathode ray oscilloscope are 1.00 cm apart. A voltage difference of 1000 V is maintained between the plates. (a) What is the electric field between the plates? (b) What force would act on an electron in this field? (c) What would be its acceleration?

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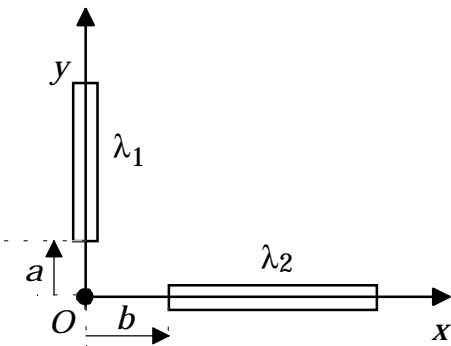


Diagram for problem 40.

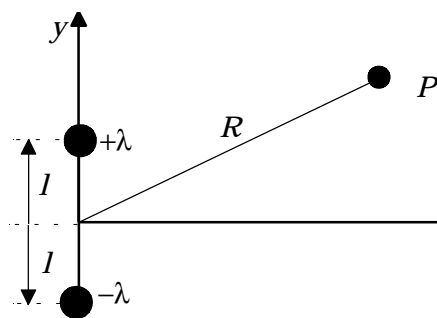


Diagram for problem 41.

43. An electron experiences an acceleration of 5.00 m/s^2 in an electric field. Find the magnitude of the electric field.

*44. An electron with an initial velocity of $1.00 \times 10^6 \text{ m/s}$ enters a region of a uniform electric field of 50.0 N/C , as shown in the diagram. How far will the electron move before coming to rest and reversing its motion?

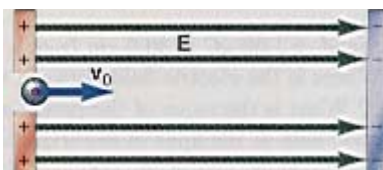


Diagram for problem 44.

45. Find the speed of an electron and a proton accelerated through a field of 200 N/C for a distance of 2.00 cm .

*46. An electron enters midway through a uniform electric field of 200 N/C at an initial velocity of 400 m/s , as shown in the diagram. If the plates are separated by a distance of 2.00 cm , how far along the x -axis will the electron hit the bottom plate.

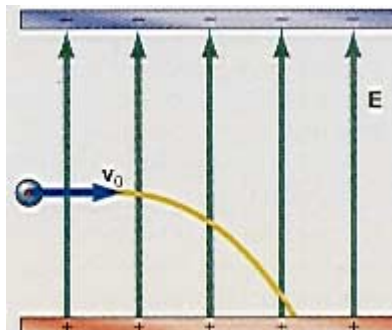


Diagram for problem 46.

Interactive Tutorials

47. *The electric field.* Calculate the value of the electric field E every meter along the line that connects the positive charges $q_1 = 2.40 \times 10^{-6}$ C located at the point (0,0) and $q_2 = 2.00 \times 10^{-6}$ C located at the point (10,0) meters.

48. *Multiple charges and the electric field.* Two charges $q_1 = 8.32 \times 10^{-6}$ C and $q_2 = -2.55 \times 10^{-6}$ C lie on the x -axis and are separated by the distance $r_{12} = 0.823$ m. (a) Find the resultant electric field at the point A , a distance $r = 0.475$ m from charge q_2 , caused by the two charges q_1 and q_2 . The line between charge 2 and the point A makes an angle $\phi = 60.0^\circ$ with respect to the positive x -axis. (b) Find the force F acting on a third charge $q = 3.87 \times 10^{-6}$ C, if it is placed at the point A . See figure 21.6 for a picture of a similar problem.

49. *The electric field of a continuous charge distribution.* A rod of charge of length $L = 0.100$ m lies on the x -axis. One end of the rod lies at the origin and the other end is on the positive x -axis. A charge $q' = 7.36 \times 10^{-6}$ C is uniformly distributed over the rod. (a) Find the electric field at the point A that lies on the x -axis at a distance $x_0 = 0.175$ m from the origin of the coordinate system. (b) Find the force F that would act on a charge $q = 2.95 \times 10^{-6}$ C when placed at the point A .

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