Lab Guide
Position, Velocity and Acceleration

I. Objectives

After completing this simulation, the student will be able to:
1. Qualitatively describe the motion of an object of known velocity and acceleration
2. Qualitatively describe the motion of an object of known position and velocity
3. Quantitatively describe the mathematical relationship between position and displacement
4. Quantitatively describe the mathematical relationship between displacement, velocity and time
5. Quantitatively describe the mathematical relationship between velocity, acceleration and time
6. Distinguish between the directions of velocity and acceleration.

II. User Interface and Simulation Features

You can change the beginning position of the person in the simulation to any point in the four regions. Simply click on and drag the person to the desired location. This is useful for changing the *distance* in which the person is affected by the acceleration value you have specified. Notice that the horizontal axis on the graph is in units of time, not position. Where the person is at any time depends on the values you have set for velocity and acceleration. The three-dimensional bar graph of the regions at the top is drawn in orthogonal projection. The person is located with respect to the lines on the horizontal top surface of the bar on which the person walks. These lines correspond to the tick marks on the number scale at the rear. When you wish to stop the simulation with the person at a specific position or at a specific time, use the “Pause” icon in the control panel at the left side of the screen.

The multiplier feature for velocity and acceleration can make a graph easier to read. For example, if you use a multiplier value of 5 for velocity, the velocity values will appear five times as great on the graph. However, the actual value used for calculation remains the same as before.

The digital values for acceleration and velocity used in calculations are controlled by your use of the analog sliders. These digital values may differ very slightly from the displayed values, which are rounded. There are situations in which velocity is zero, but the person appears to be moving very slowly. This is the visual equivalent of rounding a calculator value to the correct number of significant digits in a written exercise and does not indicate an error in your work.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Set the acceleration to zero and the initial velocity to 5.0 m/s in all four regions. Run the simulation. Does the velocity change at any point in time? According to the theoretical calculations, should the velocity change?

2. In Item 1, what was the value of the slope of the plot of position versus time?

3. In Item 1, how long did it take the person to travel 40 meters (use the “Pause” icon)?

4. Under the conditions in Item 1, how long should it take for the person to travel 25 meters? How close can you come to this value by running the simulation?

5. Set the acceleration to zero and the initial velocity to –3.0 m/s. Move the person to the division between Regions C and D. Run the simulation. How long did it take for the person to reach the left side of Region A? What was the sign of the displacement? What was the sign of the elapsed time?

<u>b. Intermediate Level</u>

6. Set the acceleration to 1.0 in all four regions and the initial velocity to 0 m/s. Run the simulation. How long does the person take to travel the 40 meters? Is the slope of velocity vs. time equal to the acceleration? Remember that the default multiplier for acceleration is 5.

7. Set the acceleration to $2.0\,\mathrm{m/s^2}$ in all 4 regions and the initial velocity to zero. Compare the time it takes the person to travel the 40 meters at this acceleration to the time in Item 6.

8. For Item 7, use the theoretical formulae to calculate the final velocity at 10 meters. Then calculate the final velocity at 40 meters. Hint: Use the final velocity at 10 meters as the initial velocity for the final 30 meters. How do these values compare with the indicated values?

9. Set the acceleration to $1.0\,\mathrm{m/s^2}$ in Regions A and B, and to $2.0\,\mathrm{m/s^2}$ in Regions C and D. Set the initial velocity to 0 and the velocity multiplier to 2.0. Run the simulation. What is the final velocity of the person indicated at 10 meters? What is the final velocity indicated at 40 meters?

<u>c. Advanced Level</u>

10. Set the initial velocity to $20\,\mathrm{m/s}$ and the acceleration in all 4 regions to $-5.0\,\mathrm{m/s^2}$. Run this simulation, which is like a car or an aircraft slowing down. How long does it take to accelerate to a stop in 40 meters? Experiment with acceleration, velocity and position. Can you find a set of parameters in which acceleration alternates between positive and negative values from Region A through Region D, the person starts somewhere in Region B or C, and the simulation goes through several cycles with the person traveling back and forth? How many such cycles can you produce? Is there a limit to the number?

Note: One such combination $v_i = 0$, a is +5.0, -2.0, +1.0 and –3.1 $\mathrm{m/s^2}$ in A, B, C and D, respectively. The initial position of the person is critical, very close to the right side of Region D. The user can usually produce 3 to 5 cycles, depending on exact initial position. With other combinations, there is no limit to the number of cycles within the 100 seconds allowed by the simulation.

Lab Guide
Free Fall Laboratory

I. Objectives

After completing this simulation, the student will be able to:
1. Qualitatively describe acceleration in free fall
2. Understand that free fall acceleration is independent of mass in a vacuum
3. Predict vertical position based on initial velocity and time
4. Qualitatively describe the factors that affect air resistance
5. Qualitatively describe the effect of air resistance on free fall
6. Quantitatively describe the effects of air density and flow on objects in free fall

II. User Interface and Simulation Features

The multiplier feature for position, velocity and acceleration can make a graph easier to read. For example, if you use a multiplier value of 5 for velocity, the velocity values will appear five times as great on the graph. However, the actual value used for calculation remains the same as before. The “delta t” slider changes the time period between points plotted on the graph. Increasing delta t also appears to increase the acceleration on the graph, but the value you provide is used for calculation and remains the same. You must explicitly stop this simulation using the icon at the left of the screen. You can turn the “bounce” on and off. A fixed coefficient of restitution for the ball is built into this simulation, so you cannot change the amount of bounce.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Accept the default values and run the simulation. What type of curve is the plot of vertical position versus time? How does this relate to the formula for motion with constant acceleration?

2. Accept the default values. Click the button to display velocity as well as position. Examine the plot of velocity versus time. Is the plot a straight line or a curved line? At what point in the ball’s motion does the velocity plot cross zero? What does this mean?

3. Accept the default values, but check the “snapshot” button and uncheck the “bounce” button. Run the simulation. In the snapshot model at the left, count the vertical scale divisions (don’t worry about actual units of measure) from the point of release to the fifth snapshot. Then do the same at the tenth snapshot. Compare these two vertical displacements. What is their ratio? From the theoretical formulae, what would you expect their ratio to be?

4. Verify that the air density is set to $0\ \mathrm{kg/m^3}$. Unclick the “bounce” button. Run the simulation. Print or record the results. Then change the radius of the ball to $0.50\ \mathrm{m}$. Run the

simulation again. Compare the results of the second run with the first run. Did the time to impact change? Why?

5. Set the variables as in Item 3. Now change the mass of the ball to 1.00 kg. Run the simulation. Did the time to impact change compared to the two runs in Item 3? Why?

b. Intermediate Level

6. Set the air density to 1.3 kg/m^3. Check the velocity display button. Run the simulation. Examine the plot of velocity versus time. Is this plot a straight line or a curve? What does this mean in terms of acceleration?

7. Set the air density to 1.3 kg/m^3. Check the velocity display button. Change the radius of the ball to 0.50 m. Run the simulation. Compare the velocity plot to the one in Item 6. Is there a difference? Why?

8. Set the air density to 1.3 kg/m^3. Check the velocity display button. Change the radius of the ball to 0.50 m and the mass of the ball to 1.00 kg. Run the simulation. Compare the velocity plot to the ones in Items 6 and 7. Is there a difference? Why?

c. Advanced Level

9. This simulation is not intended to model the motion of the ball when it is in contact with the ground surface. How do the velocity and acceleration plots show this?

10. Terminal velocity is reached when the force of air resistance is equal to the force of gravity. Set the radius of the ball to 0.50 m and air density to 1.3 kg/m^3. Click the display buttons for position, velocity and acceleration. Run the simulation. Now change the mass of the ball to find the lowest mass that will result in a terminal velocity before the ball strikes the ground. What value did you find for this mass?

11. Set the radius of the ball to 0.50 m, the air density to 1.3 kg/m^3, and the wind speed to 13 m/s upward. Find a value for the mass of the ball for which the force of gravity is just equal to the force of air resistance caused by the wind.

Lab Guide
Air Track

I. Objectives

After completing this simulation, the student will be able to:
1. Calculate final velocity in an elastic collision.
2. Calculate final velocity in an inelastic collision.
3. Compare and contrast conservation of momentum and energy in collisions.
4. Describe elastic and inelastic collisions in terms of coefficient of restitution.

II. User Interface and Simulation Features

This simulation consists of carts colliding in one-dimensional motion on a frictionless air track. You can vary the coefficient of restitution for these collisions. A value of 0 represents a completely inelastic collision and a value of 1.0 represents a completely elastic collision. The mass and velocity of each cart can be changed. You can also drag each cart to a desired starting position before starting the simulation. You can choose to show the location of the center of mass of the two-cart system. The simulation continues to run after the carts reach the ends of the track so that you can record or print the final momentum and velocity values. When you are done, remember to stop the simulation using the icon on the left of the screen.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. From the default values, calculate the initial momentum of each cart. Be careful to use the correct sign, since momentum is a vector quantity. What are these values? Algebraically add the two values. Run the simulation. What is the final total momentum value indicated by the simulation? Does it differ from the value you calculated?

2. Accept the default values. Change the velocity of cart B to $-6.0\ \mathrm{m/s}$. Calculate the final velocity of this inelastic collision. Run the simulation. Does the value for final velocity in the simulation agree with your value?

3. Set the velocity of cart A to $0\ \mathrm{m/s}$. Set the initial velocity of cart B to $-6.0\,\mathrm{m/s}$. Run the simulation. Record or print the final velocity value. Now change the mass of cart B to $6.0\ \mathrm{kg}$, equal to that of cart A. Run the simulation. Compare the final velocity to the previous run. Now find the ratio of the final velocity of this second run to the initial velocity of cart B. Why should this ratio have this value?

4. Place cart B in the middle of the track and set its velocity to $2.0\,\mathrm{m/s}$. Set the velocity of cart A to $6.0\ \mathrm{m/s}$. Run the simulation. What is the value of the final velocity?

<u>b. Intermediate Level</u>

5. Accept the default values, but change the coefficient of restitution to 1.00. Run the simulation. What happens to the carts? Is the final momentum different than the final momentum in Item 4? Why should this be or not be?

6. Set the mass of each cart to $5.0\ \mathrm{kg}$. Set the velocity of cart A to $6.0\,\mathrm{m/s}$ and the velocity of cart B to $-6.0\,\mathrm{m/s}$. Run the simulation. What happens to the carts? What are the values of the initial and final total momenta? Change the coefficient of restitution from 0 to 1.00. Run the simulation again. What happens to the carts? Does the value of final total momentum change from the previous run?

<u>c. Advanced Level</u>

7. Calculate the initial and final total kinetic energy for the system of the two carts in Item 4. Now set the values as in Item 4, but change the coefficient of restitution to 1.00, making it a perfectly elastic collision. Run the simulation again, and calculate the initial and final total kinetic energies. How do these compare with those from Item 4?

8. Accept the default values. Run the simulation five times, changing the coefficient of restitution to the values 0.00, 0.25, 0.50, 0.75 and 1.00 with all other values remaining constant. After each run, print or record the final velocities. Plot the value for the decrease in kinetic energy (from the maximum value) against coefficient of restitution. What do you think the shape of this curve is? What do you think happens to the kinetic energy that is “lost”, considering the law of conservation of energy?

Lab Guide
Projectile Motion

I. Objectives

After completing this simulation, the student will be able to:
1. Describe the shape of the path of an object in projectile motion
2. Describe the relative direction of the velocity and acceleration vectors for an object in projectile motion at different points in its path.
3. Compare the two angles that will result in the same *x*-displacement value
4. Compare and contrast projectiles launched horizontally and projectiles launched at an angle.

II. User Interface and Simulation Features

The default conditions include the display of the initial velocity vector, $\vec{v}_i$ (in blue). There are two ways of changing the initial angle θ_i and $\vec{v}_i$ of the object. With the simulation stopped, you can enter values for these variables in the Launch Settings, or you can enter values for the *x*- and *y*-components of the initial velocity, which will accomplish the same thing. After you have entered values in the Launch Settings boxes, you must touch the <enter> key to accept the values. As an alternative, you can drag the tip of the $\vec{v}_i$ vector to its desired magnitude and direction. This second method does not require you to touch the <enter> key. If you wish to start the simulation with the object in a different position, move the object (and the tail of

There are two ranges for the coordinate system. With the Graph Limit Range button checked, each square on the grid represents one meter, with a maximum displayed range of 37 meters. With the Graph Limit Range button unchecked, each square represents 3 meters with a maximum range of 111 meters. The *x*- and *y*- axes cannot be set independently of each other.

The Bounce On/Off button provides a perfectly elastic collision between the object and the “walls” of the two axis and the range limits.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Accept the default values and run the simulation. Stop the simulation; uncheck the Bounce On/Off button. Run the simulation again. Stop, then uncheck the Graph Limit Range button. Run the simulation again. Examine the theoretical formulas for *y*- and *x*- position. What is the shape of the path of the object? According to the formulas, what shape should it be?

2. Turn off the Bounce. Move the object to a height of 30 meters on the *y*-axis. Set v_i to $10.0\,\mathrm{m/s}$ horizontally ($\theta = 0$). Run the simulation. Record or print the values for $v_{f,y}$, y, and t.

With the object remaining at 30 meters, run the simulation with $v_i = 5.0\,\text{m/s}$ and $v_i = 15.0\,\text{m/s}$, recording the final values each time. Compare the final values for the three runs. What values change? What values remain the same?

3. Now move the object to the point $(37,30)$ on the grid. Repeat the simulations as in Item 2, but use negative velocities, i.e., $-5.0\,\text{m/s}, -10.0\,\text{m/s}$, and $-15.0\,\text{m/s}$ directed horizontally. Compare and contrast these results with those in Item 2.

4. Position the object at the point $(5,30)$ on the *y*-axis. What initial velocities would you use to change projectile motion into free-fall motion?

b. Intermediate Level

5. Position the object at the point $(0,30)$ on the *y*-axis. Set $v_x = 3\,\text{m/s}$ and $v_{0,y} = -11.0\,\text{m/s}$. Run the simulation. Why does the object bounce higher than the starting position? What velocity values will allow the object to bounce exactly as high as the starting position?

6. Position the object at the point $(0,0)$. Accept the default values, but turn off Bounce. Run the simulation and record or print the final x-position. Leaving all other Launch Settings constant, what angle will give you the same final x-position? What relationship is there between these two angles?

7. Think about the answers to Item 6 and what happens when you slowly decrease the default angle of $71.6°$? What angle has the same relationship to *itself* that the two angles in Item 6 have to each other? Holding all other Launch Settings constant, what angle θ_0 gives the maximum displacement in the *x*-direction?

c. Advanced Level

8. After first having completed Item 7, move the object to the point $(0,20)$. Turn off Bounce. Set $v_0 = 6.0\,\text{m/s}$ and $\theta_0 = 0°$. Run the simulation and note the final *x*-position. Keeping all other Launch Settings constant, change θ_0 to $45°$. Run the simulation again, and compare the final *x*-position with that of the previous run. Why did you obtain the results you did here?

9. Move the object to the point $(37,30)$ on the grid. Verify that Bounce is turned on. Are you able to find a value for v_0 that will cause the object to continue a repetitive pattern, bouncing at the bottom center of the grid, i.e., at the point $(18.5,0)$? Does it help to turn off the Grid Snap? What do you think causes the results that you observe?

Lab Guide
Vector Properties

I. Objectives

After completing this simulation, the student will be able to:
1. Describe a vector as representing a quantity that has magnitude and direction
2. Explain that the tip, or head, of a vector represents its direction, and that the length of a vector represents its magnitude.
3. Show how to resolve a vector into its *x- and y-*components.
4. Show how to add perpendicular vectors graphically and numerically.
5. Show how to add non-perpendicular vectors graphically and numerically.
6. Use vector space notation in two dimensions (2-space).

II. User Interface and Simulation Features

Each scale division on the grid represents two meters. You can choose to work with a number of vectors from one to four by selecting from the slider bar at the top of the screen. These vectors are labeled A, B, C and D at their midpoints. The Grid Snap function restricts the tail and the tip of the vector to integral positions, in meters, on the grid. You can change the length and direction of the colored vectors that you select by dragging the tail or the tip of the vector. If you want to move, or translate, the vector without changing its length or direction, drag it by its center label. The black vector is labeled Σ to indicate that it is the resultant vector of the other visible vectors. You cannot change its length and direction directly. You can move it by dragging it by the label.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Select 1 vector using the slider. Compare the magnitude and direction of the black resultant vector with those of the single blue vector "A"? Drag the tip of the blue vector around to change its magnitude and direction. Now drag the tail of the blue vector around to change its magnitude and direction. In each case, what happens to the black resultant vector?

2. In the case of Item 1, can you cause the blue vector and the resultant black vector intersect at an angle? Why or why not?

3. Select 2 vectors using the slider. Translate the blue A vector so that its tail is at the origin and it is horizontal with a length of 20 meters. Translate the red B vector so that its tail is at the tip of the blue A vector, and it is vertical with a length of 10 meters. Notice that the red and blue vectors form a right angle. Now translate the resultant vector so that its tail is at the same point as the tail of the blue A vector, and its tip is at the same point as the tip of the red B vector. Use the Pythagorean formula to calculate the length of the hypotenuse of this right triangle. What

value do you obtain? The blue vector is the *x*-component of the resultant, and the red vector is the *y*-component of the resultant. Now translate the resultant vector to another position on the grid. What effect does this have on the blue feedback values in the upper right corner of the screen?

4. Select 2 vectors with the slider. Translate the black resultant vector so that its tail is at the origin. Adjust the direction of the blue A vector so that it points horizontally to the right. Adjust its length so that the tip of the **resultant** vector is 30 meters horizontally from the y-axis. Adjust the direction of the red B vector so that it is vertical upward. Adjust its length so that the tip of the **resultant** vector is 20 meters above the *x*-axis. Must the x- and y-components of the resultant actually be drawn to form the sides of a right triangle? Confirm with the Pythagorean formula that the blue and red vectors are the *x*- and *y*-components of the resultant.

b. Intermediate Level

5. Select 2 vectors using the slider. Turn the Grid Snap off. Position the blue A vector so that its tail is at the origin and its tip points at a $45°$ angle below the negative *x*-axis with a length of 14.4m. Position the red B vector so that its tail is at the tip of the blue vector and its tip points at an angle of $45°$ below the positive *x*-axis, also with a length of 14.4m. Notice that the blue and red vectors form a right triangle. How do you know this? Using trigonometry and the lengths of these two vectors, calculate the x- and y-components of each of the blue and red vectors. Algebraically add the two *x*-components. Algebraically add the two *y*-components. These two sums give the *x*- and *y*-components of the resultant vector. Do your values agree with those in the simulation?

6. Create the *x*- and *y*-component vectors of the resultant vector by selecting 2 vectors using the slider. Position the tail of blue vector A at the origin. Now position the tail of the resultant at the origin. Drag the tip of vector A in the negative *x*-direction so that it has a length of 24 meters. Now drag the tail of red vector B to coincide with the tip of vector A. What has happened to the position of the resultant vector? Calculate the length of the resultant vector.

7. Select 2 vectors with the slider. Position the blue A vector so that its tail is at the origin and it has a length of 10 meters coincident with the positive x-axis. Now position the red B vector so that it is also coincident with the A vector, but with a length of 15 meters. Notice the value of the dot product in the blue feedback values in the upper right corner of the screen. The dot product of two vectors $\vec{A}$ and $\vec{B}$ is the product of the magnitudes of the two vectors multiplied by the cosine of the angle between them, or $\vec{A}\cdot\vec{B} = AB\cos\theta$. Move the tip of the blue vector slowly until it is coincident with the y-axis. What happens to the dot product? Now slowly move the tip of vector B counterclockwise. In what regions is the dot product positive? Where is it negative? Where is it zero?

c. Advanced Level

8. Repeat Item 7, but initially place both vectors A and B pointing $45°$ above the negative axis. Drag the tip of vector B counterclockwise as before. How does the change in the sign of the dot product differ from its change in Item 7? Consider the definition of the dot product. Why does the change in sign differ in the way it does? Notice that you can only calculate the dot product of two vectors.

9. Select 4 vectors using the slider. Position them arbitrarily, but separated, on the grid, with random lengths and directions. Lengths of 10 meters or less work best here. Calculate the x- and y-components of each vector and calculate the coordinates of the resultant vector by summing. Do your values agree with the displayed values? Now carefully translate vector A, dragging by its label, so that its tail coincides with that of the resultant. Position each of the other 3 vectors, tail-to-tip in the same way. What is the location of the 4^{th} vector with respect to the resultant vector?

Lab Guide
Centripetal Force

I. Objectives

After completing this simulation, the student will be able to:
1. Describe the *x*- and *y*-displacement of an object in circular motion in each of the four quadrants
2. Describe the relationship between acceleration on one axis and instantaneous velocity on the other.
3. Describe the effect of change in the mass of an object on its centripetal force.
4. Describe the effect of change in the radius of circular motion on centripetal force.
5. Describe the effect of change in the instantaneous velocity of the object on centripetal force.

II. User Interface and Simulation Features

This simulation shows an object in rotational motion around a central point. The mass and instantaneous tangential velocity of the object can be varied, as can the radius of angular motion. Vectors representing centripetal force on the object (in green) and instantaneous velocity of the object (in blue) are shown. Output variables of centripetal force and instantaneous angular velocity are shown on the left.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Accept the default values, but set instantaneous velocity to $20\,\mathrm{m/s}$. Run the simulation. Observe how the *x*-position changes with respect to *y*-position. By how many degrees is the *x*-position out of phase with the *y*-position? What is this phase difference expressed in radians? In which quadrant is x negative and y positive? In which quadrant are x and y both negative?

2. Accept the default values, but set instantaneous velocity to $20\,\mathrm{m/s}$. Run the simulation. What is the relationship between $x\text{-position}$ and v_y? Compare and contrast this relationship with the one between $y\text{-position}$ and v_x. What are the two phase angles?

3. Accept the default values, but set instantaneous velocity to $20\,\mathrm{m/s}$. Run the simulation. What is the value of v_y when a_y is at its maximum value? Compare and contrast this with velocity and acceleration in the *x*-direction.

4. Accept the default values and run the simulation. Record the value for centripetal force. Stop the simulation. Now change the mass to $10.0\ \mathrm{kg}$. Run the simulation. What is the ratio of the

new centripetal force to the old value? Compare this to the ratio of the new mass value to the previous mass value.

5. Accept the default values. Predict what the value will be for centripetal force if you change the mass from $1.0\ \mathrm{kg}$ to $5.0\ \mathrm{kg}$. Run the simulation. Is your value the same as that shown in the simulation?

b. Intermediate Level

6. Change the default value for radius to $50.0\ \mathrm{m}$. Run the simulation. Record the value for F_c. Stop the simulation. Now change the value for radius to $100.\ \mathrm{m}$. Run the simulation again. Record the value for F_c. What is the ratio of the new to old radius values? What is the ratio of new to old values for $F_{c.}$? Is this what you would expect from the theoretical equations?

7. Change the default value for instantaneous velocity to $50.0\ \mathrm{m/s}$. Run the simulation. Record the value for F_c. Stop the simulation. Now change the value for instantaneous velocity to $100.\ \mathrm{m/s}$. Run the simulation again. Record the value for F_c. What is the ratio of the new to old velocity values? What is the ratio of new to old values for $F_{c.}$? Is this what you would expect from the theoretical equations?

c. Advanced Level

8. Predict the value for F_c when default values are used, except radius is chosen to be $25\ \mathrm{m}$. Run the simulation to check your answer.
9. Predict the value for F_c when default values are used, except instantaneous velocity is $75\ \mathrm{m}/s$.

Lab Guide
Force Table

I. Objectives

After completing this simulation, the student will be able to:
1. Define net force as the vector sum of forces acting on an object.
2. Describe net force in terms of Newton's second law of motion
3. Estimate net force, given one to three force vectors,
4. Predict the conditions for equilibrium by adding force vectors.

II. User Interface and Simulation Features

This simulation represents the user looking down vertically on a force table. A frictionless ring on top of the table is connected to four strings. Each string can exert a force of zero to 10 N on the ring by means of sliders in the upper left part of the screen. The direction in which the force is applied can also be changed by means of sliders above the screen. The positive *x*-axis represents zero degrees, as in a unit circle. In practice, the forces are applied by masses attached to the other ends of the four strings, hanging over low-friction pulleys. Each force is labeled by subscripts from 1 to 4. When the net force applied to the ring is zero, the color of the axes changes to yellow.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Set all forces to zero. Set $F_1 = 5.0\text{ N at }0°$. Observe the color of the axes and the net force. Record the net force and direction. "Pull the pin" by starting the simulation. Observe what happens to the location of the ring. Why does this happen?

2. Set all forces to zero. Set $F_1 = 5.0\text{ N at }0°$ .and $F_2 = 5.0\text{ N at }180°$. Observe the color of the axes and the net force. Record the net force and direction. Run the simulation. Observe what happens to the location of the ring. Why does this happen?

3. Set all forces to zero. Set $F_1 = 5.0\text{ N at }0°$, $F_2 = 5.0\text{ N at }90°$ and $F_3 = 5.0\text{ N at }270°$. Compare the net force and direction to that of Item 1. Run the simulation. What happens to the location of the ring? Is the result different from that of Item 1? Why?

<u>b. Intermediate Level</u>

4. Set all forces to zero. Set $F_1 = 5.0\text{ N at }0°$ and $F_2 = 8.0\text{ N at }180°$. Observe the color of the axes and the net force. Record the net force and direction. Run the simulation. Observe what happens to the location of the ring. Allow the ring to come to a stop. Repeat this by maintaining

F_1 constant and using values of $F_2 = 6.0\text{ N at }180°$, $F_2 = 5.2\text{ N at }180°$ and $F_2 = 5.1\text{ N at }180°$. Observe the similarity or difference between these three trials. The simulation has a tolerance for error of about 0.2 N. This means that the net force must equal at least 0.2 N before the simulation will indicate that the forces are not balanced. However, the ring will still react to the actual net force. Based on your observation, do you think the tolerance is too high? What do you think would happen if the tolerance was only 0.05 N

5. Set all forces to zero. Set $F_1 = 7.0\text{ N at }0°$, $F_2 = 7.0\text{ N at }90°$, $F_3 = 7.0\text{ N at }180°$ and add a fourth force, $F_4 = 5.0\text{ N at }270°$. Run the simulation. What happens to the ring? Now stop the simulation, set $F_4 = 9.0\text{ N at }270°$ and run the simulation again. What is different about the final location of the ring this time?

<u>c. Advanced Level</u>

6. Set $F_1 = 5.0\text{ N at }0°$, $F_2 = 5.0\text{ N at }90°$ and $F_3 = 5.0\text{ N at }270°$. Run the simulation. What force in the negative *x*-direction would be necessary to produce a net force of zero when added to F_1? Using this information, estimate the *x*-components of F_2 and F_3. If F_2 and F_3 each represent the hypotenuse of a right triangle, what is the angle made by F_2 and F_3 with the *y*-axis?

7. Set $F_1 = 5.0\text{ N at }0°$, $F_2 = 5.0\text{ N at }90°$ and $F_3 = 5.0\text{ N at }270°$. Using trigonometry, what net force of F_4 in the negative *x*-direction is necessary to produce an angle of $15°$ between F_2 or F_3 and the *y*-axis? Set F_4 to that value and run the simulation. Does the angle formed approximate $15°$?

8. Set $F_1 = 5.0\text{ N at }0°$, $F_2 = 5.0\text{ N at }90°$, $F_3 = 5.0\text{ N at }180°$ and $F_4 = 5.0\text{ N at }315°$. Can you predict the final position of the ring? Run the simulation. Observe the results. Given the magnitudes of the four forces, what other information would you need to calculate the actual position of the ring at equilibrium? How would you use this information?

Lab Guide
Mass on Inclined Plane

I. Objectives

After completing this simulation, the student will be able to:
1. Explain the law of conservation of energy as it applies to kinetic and potential energy
2. Describe the effect of angle on the acceleration of a mass down an inclined plane
3. Describe the effect of mass on acceleration and velocity of a mass on an inclined plane
4. Describe the effect of friction on the velocity and acceleration of a mass on an inclined plane

II. User Interface and Simulation Features

In this simulation, a massive block is situated on an inclined plane. The angle of the plane with horizontal can be varied, and the block can initially move up the plane or down the plane. A negative velocity value implies velocity up the plane. The mass of the block and the coefficient of friction between the block and the plane can also be varied. You can plot any of the variables in the lower right corner of the screen against any other one of these variables. To do so, drag the names of the variables into the boxes labeled "x choice" and "y choice" on the graph. Each axis has a variable range of 0-5 units through 0-1000 units. Change the range by clicking on the +/- icon next to the current range value.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Accept the default values, but set the y-axis to 500J. Plot total energy E on the *y*-axis versus time on the *x*-axis. Notice that the coefficient of friction is zero. Run the simulation. What do you observe? What law does your observation illustrate?

2. Change the angle to $90°$. Plot y vs. t. Run the simulation. What type of motion does this special case represent?

3. Set the angle to $30°$. Run the simulation. Using the theoretical formulas, what should acceleration down the incline, a_x be? What value does the simulation report for a in the upper right of the screen? Does this value change during the simulation?

4. Set the angle to $30°$ and change the mass to 10.0 kg. Plot a_x versus t. Run the simulation. Does the acceleration in the *x*-direction depend on the mass? Should acceleration depend on mass, according to the theoretical formulas?

b. Intermediate Level

5. Set the angle to $30°$. Plot v_y versus v_x. Set both axes to $10\ \mathrm{m/s}$. Run the simulation. Why is the plot a straight line? Give an expression for the slope of this line in terms of trigonometric functions.

6. Set the angle to $27°$ and the coefficient of friction to 0.50. Plot v_x versus time. Run the simulation. After 15 seconds, pause the simulation and print the graph. Observe the value for a_x. Remember that in this simulation, the coefficient of kinetic friction is equal to the coefficient of static friction. Now stop the simulation, reduce the angle to $26°$ and run the simulation again. After 15 seconds, pause the simulation and print the graph. What is the value for a_x?

7. Set the angle to $26°$, the coefficient of friction to 0.50, and the velocity down the plane to $-3.0\ \mathrm{m/s}$.. Plot v_x versus time. Run the simulation. After 15 seconds, pause the simulation and plot the graph. What happened to the velocity of the block? Why did this happen?

c. Advanced Level

8. Set the angle to $20.0°$ and the coefficient of friction to $\mu = 0.20$. Plot total energy E versus time, t. Set $y_{\max} = 500\mathrm{J}$. Run the simulation. After E reaches a stable value, stop the simulation and print the graph, What do you think happens to the energy that is lost from the block? Notice the shape of the curve.

9. Set the angle to $20.0°$ and the coefficient of friction to $\mu = 0.20$. Plot kinetic energy KE versus time, t. Set $y_{\max} = 500$. Run the simulation. After KE reaches a stable value, stop the simulation and print the graph. Compare this graph to the graph of total energy. Explain what happens to E and KE just before and just after the block “bounces”.

10. Set the angle to $20.0°$ and the coefficient of friction to $\mu = 0.20$. Plot potential energy PE versus time, t. Set $y_{\max} = 500$. Run the simulation. After PE reaches a stable value, stop the simulation and print the graph. Compare this graph to the graph of total energy. Explain what happens to E and PE just before and just after the block “bounces”.

Lab Guide
Center of Mass

I. Objectives

After completing this simulation, the student will be able to:
1. Describe the meaning of "center of mass"
2. Describe the meaning of "axis of rotation"
3. Explain what happens to moment of inertia when the mass is increased
4. Explain what happens to moment of inertia when distance is increased
5. Use the parallel-axis theorem to calculate moment of inertia around any axis of rotation.
6. Estimate center of gravity and moment of inertia for various systems of mass and distance.

II. User Interface and Simulation Features

This simulation allows you to place masses of 50 grams each at locations in a Cartesian coordinate system. Each mass is represented by a yellow square which can be dragged to the desired location on the grid. Each line on the grid represents a distance of 4 cm. More than one mass can be placed in the same location. If you do this, a black number appears telling you how many masses are "stacked up" at this location. You can place the masses at any location you like by unchecking the Grid Snap On/Off button. Make certain you place the <u>center</u> of each mass at the location you intend. The red crosshairs indicate the location of the center of mass of all masses on the grid. The blue crosshairs represent the location of the axis of rotation and can be located wherever you want on the grid. The positions of the cursor, center of mass and rotational axis are shown in units of cm in the lower left corner of the screen. The moment of inertia, I, is shown in units of $\mathrm{g \cdot cm^2}$ up to a value of 100 000, when the units change to $\mathrm{kg \cdot m^2}$. In the lower right corner of the screen are several preset systems of mass from which you can choose.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Place one mass of 50 g on the grid at the location $(20, 0)$. Where is the center of mass?

2. Place one mass at the location $(20, 0)$ and another at the location $(-20, 0)$. Where is the center of mass located?

3. Now place two masses at the location $(20, 0)$ and two masses at the location $(-20, 0)$. Where is the center of mass located? Compare this to your result in Item 2. Is the location of the center of mass dependent on the magnitude of the masses?

4. Place two masses at the location $(20,0)$ and two masses at the location $(-20,0)$. Where is the center of mass located? Now move the two masses from $(-20,0)$ to $(-16,12)$. Is the location of the center of mass dependent on the positional distribution of the masses?

5. Place one mass at the location $(20,0)$ and three masses at the location $(-20,0)$. Where is the center of mass located? Express the distance of the center of mass from the smaller mass as a fraction of the total distance between masses. How does this compare to the ratio of the larger mass to the total mass in the system?

b. Intermediate Level

6. Place one mass at the origin, $(0,0)$. Place the axis of rotation at $(4,0)$.Using the theoretical equations, what should the moment of inertia, I , be around the axis of rotation? What does the simulation report I to be?

7. Place one mass at the origin, $(0,0)$. Place the axis of rotation at $(4,0)$.Observe the reported value for I . Now move the axis of rotation to $(8,0)$, doubling the distance from the mass to the axis of rotation. How does the value of I change? Move the axis of rotation to $(16,0)$, doubling the distance again. What is the value of I now? What is the relationship between mass, distance to axis of rotation, and I ?

8. Place one mass at $(0,-20)$ and another mass at $(0,20)$. Position the axis of rotation at the origin. Observe the reported value for I . Do the moments of inertia of the two masses cancel each other? Why not?

9. Select the center preset at the lower right corner of the screen, consisting of a line of seven individual masses. Observe where the center of mass is located. Locate the axis of rotation on the center of mass. Using the theoretical equations, calculate the value of I . Does your value agree with the value reported by the simulation?

10. Repeat Item 9, but locate the axis of rotation at the end of the line of masses, $(14,0)$. Why is this value for I larger than that in Item 8?

c. Advanced Level

11. Using the results of Item 9, express the answer to Item 10 according to the parallel-axis theorem and calculate the result. Does the answer to Item 10 agree with your answer here?

12. Place one mass each at $(0,-20)$, $(0,20)$, $(20,0)$ and $(-20,0)$. Position the axis of rotation at the origin. How does the value for I compare to that of Item 8? If you had a large number of 50 gram masses of total mass M distributed equidistantly from the axis of rotation, what single formula could be used to find the moment of inertia of this shape? What would you call this shape? Give a real-life example of an object that has this approximate shape.

13. Suppose you had two masses, one of 1000 g and the other of 100 g. You wish to reduce the moment of inertia to a minimum value. To accomplish this, where would you locate the axis of rotation relative to the center of mass? Express the distance from the smaller mass to the center of mass as a fraction of the total distance separating the two masses.

Lab Guide
Two Mass Oscillator

I. Objectives

After completing this simulation, the student will be able to:
1. Describe the motion of an oscillating system
2. Describe the effect of increasing drag on an oscillating system
3. Find the natural frequency of a two-mass oscillator
4. Calculate the natural frequency of a two-mass oscillator.
5. Find the two resonant states of a two-mass oscillator

II. User Interface and Simulation Features

This simulation demonstrates the motion of a system consisting of two masses connected by a spring. The mass at the right is connected to a stationary block by a second spring. The mass at the left is connected to a block that you may choose to be either stationary or the source of a driving oscillation of variable frequency. The initial positions of the two masses may be varied by dragging them. The driving oscillator may be turned on or off by checking a button. The spring constants of the springs and the magnitudes of the two masses may be individually varied by the means of the labeled sliders. All of these variables must be set before running the simulation. The driving frequency, however, may be varied <u>during</u> a run of the simulation.

Remember that the natural frequency of an oscillator, $\omega_0 = \sqrt{\frac{k}{m}}$ is in units of radians/second.

The frequency in hertz is $f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Accept the default values. Run the simulation. Does the motion of the masses remain constant, or does it decay, or decrease in amplitude?

2. Change the drag coefficient to its minimum value of $0.1\ \text{kg/s}$. Run the simulation. Stop the simulation. Increase the drag coefficient to $0.4\ \text{kg/s}$. Run the simulation again. What do you observe that is different about the decay of the motion? Stop the simulation and increase the drag coefficient to $1.0\ \text{kg/s}$. Run the simulation again. What effect does the drag coefficient have on the motion of the two masses?

3. Run the simulation. How long does it take for the system to come nearly to rest? Stop the simulation and drag each block as far to the right as possible. Run the simulation again. Observe the time the system takes to come nearly to rest. Stop the simulation. Now drag the

blocks as far apart as possible. Run the simulation again. Does the initial position of the blocks have a significant effect on the decay of the motion?

4. Set the masses to $0.1\ \mathrm{kg}$ each. Run the simulation. How long does it take for the system to come nearly to rest? Stop the simulation. Now set each mass to $10\ \mathrm{kg}$. Run the simulation again. What effect does increasing mass have on the decay of the motion?

<u>b. Intermediate Level</u>

5. Turn the oscillator on by checking the Oscillator Off/On button. Set the frequency to $0.15\ \mathrm{Hz}$. Set the drag coefficient to $0.1\ \mathrm{kg/s}$. Run the simulation. Observe the motion of the masses. Increase the frequency to $0.30\ \mathrm{Hz}$. What change in the motion do you observe? Increase the frequency to $0.40\ \mathrm{Hz}$. Now what change do you observe?

6. Turn the oscillator on. Set the frequency to $0.30\ \mathrm{Hz}$. Set the drag coefficient to $0.1\ \mathrm{kg/s}$. Run the simulation. Now increase the frequency gradually by increments of $0.01\ \mathrm{Hz}$ up to $0.40\ \mathrm{Hz}$. Be sure to allow enough time between changes for the system to come to equilibrium. What do you observe?

7. Repeat Item 6, but set the drag coefficient to $1.0\,\mathrm{kg/s}$. Run the simulation. What do you observe about the amplitude of motion compared to Item 6?

<u>c. Advanced Level</u>

8. In a two mass system, there are two resonant frequencies, one with the motion of the masses in phase, and the other out of phase. Try to find the out-of-phase resonant frequency. What value do you obtain, in Hz?

9. If you added more masses, all connected by springs, do you think there could be additional resonant states?

10. Use the mass and spring constant that you used in item 6 in the theoretical equations to calculate the angular frequency, ω_0 in rad/s., From that value, calculate the periodic frequency in Hz. Compare this value with the in-phase resonant frequency you found in Item 6.

Lab Guide
Connected Masses on Two Inclines

I. Objectives

After completing this simulation, the student will be able to:
1. Identify the components of the force of gravity, F_g , that are parallel and normal to an inclined plane.
2. Describe the effect on acceleration of two opposing force components on a system of two masses.
3. Describe the effect of the rotational inertia of a massive pulley on acceleration
4. Describe the effects of kinetic friction between the surfaces of inclined planes and the masses sliding on them.
5. Distinguish between the effects of kinetic friction and static friction.

II. User Interface and Simulation Features

This is a versatile simulation of two masses, connected by a fixed length of string. Each mass is situated on a separate inclined plane. The magnitude of the masses and the angles the plane make with horizontal can be varied. The coefficients of static and kinetic friction can be varied.. The string connecting the masses runs over a pulley. The mass of the pulley can be varied. All of these variables are changed by means of slider bars. With certain combinations of variables, the force of static friction exceeds the net force of gravity down a plane, and no motion occurs. You may see a “Nudge” button when this happens. It is located in the lower center of the screen. Clicking on this button “nudges” the apparatus, overcoming static friction. However, sometimes even a nudge will not cause the masses to move.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Use trigonometry to verify the parallel and normal components of the force of gravity on Mass 1 and Mass 2. What functions will you use?

2. Accept the default values. Run the simulation. “Nudge” the system. What is the magnitude of acceleration of each mass that you observe? Are the signs of acceleration for the two masses the same or opposite?

3. Adjust the value of Mass 1 so that the magnitude of acceleration is as close as possible to $1.00\,\mathrm{m/s^2}$. What mass value did you use? What was the actual magnitude of the acceleration? Did you still have to nudge the system?

4. Set $\text{Mass 1} = 10.0\ \text{kg}$ and $\text{Mass 2} = 0.5\ \text{kg}$. Run the simulation. Do you still have to nudge the system? Stop the simulation. Set the coefficient of kinetic friction to 0.25 and the coefficient of static friction to 0.50 for Mass 1. Run the simulation again. What is the value for acceleration?

b. Intermediate Level

5. Set the value of Mass 2 to the value you found in Item 3. Which coefficient of friction do you think is the factor requiring you to nudge the system? Find the maximum value for this coefficient that does not require a nudge.

6. In Section II, we said that sometimes even a nudge will not cause the masses to move. Describe a situation in which this might occur.

7. Set all the coefficients of friction to zero. Run the simulation. Observe the magnitude of acceleration. Stop the simulation. Increase the mass of the pulley to $8.0\ \text{kg}$. Run the simulation again. What change do you observe in the acceleration?

c. Advanced Level

8. Set all the coefficients of friction to zero. Run the simulation. Using the theoretical formula for the pulley and the default mass of the pulley, calculate the linear acceleration. Compare this value to the value reported by the simulation.

9. The simulation allows you to change the mass of the pulley, but not the radius. Why do you think this is so?

10. Build an Atwood machine! Set both angles to $\theta = 90°$. Change the mass of the pulley so that you obtain a maximum acceleration value. What is the maximum magnitude of acceleration you observe, and what is the mass of the pulley? Why does increasing the coefficient of kinetic friction have no effect here? If you set the mass of one of the masses to zero, what is the acceleration?

Lab Guide
2D Collisions

I. Objectives

After completing this simulation, the student will be able to:
1. Describe conservation of momentum in elastic collisions
2. Describe conservation of momentum in inelastic collisions
3. Describe the path of the center of mass of two colliding objects
4. Explain the relationship between conservation of energy in a collision and coefficient of restitution.
5. Explain conversions between kinetic, gravitational potential and magnetic potential energy

II. User Interface and Simulation Features

This simulation shows a plan view of (looking down upon) a square, plane, frictionless table, 32 cm on a side, upon which are two disks, or pucks. The masses of the pucks can be individually varied. Each puck has an initial velocity, represented by a vector. The initial vector magnitude and direction can be changed by dragging the tip of the vector. The initial position of the puck can be changed by dragging the puck. Collisions between the pucks can be made completely elastic, completely inelastic, and somewhere in between by varying the coefficient of restitution, ε between 0 and 1.0. ε is defined as the ratio of relative final and initial speeds of the pucks. It must be used for head-on collisions only. Each puck loses a fixed amount of kinetic energy during each collision with a wall. The pucks can be made magnetically repulsive or non-magnetic. The table can be raised at an angle. The front (bottom of the screen) of the table remains at ground level, and the rear (top of the screen) can be raised to a maximum of a 10 degree angle with the ground.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Run the simulation. Do the pucks always maintain the same speed? Why is this?

2. Record the initial *x*- and *y*-components of velocity for both the red and blue pucks. Turn on CM Trails. Run the simulation. After the first collision between pucks, pause the simulation and record the final velocity components for each puck. Calculate the *x*- and *y*- components of the initial and final momenta of each puck. What was the net momentum change in the *x*-direction? In the *y*-direction? Was momentum conserved?

3. Drag the blue puck $14\ \text{cm}$ to the left of the origin (crosshairs) on the *x*-axis. Set its velocity to $10\ \text{cm/s}$. Drag the red puck $14\ \text{cm}$ to the right of the origin on the *x*-axis. Set its velocity to $-10\ \text{cm/s}$. Change the coefficient of restitution to zero. Run the simulation. What do you

observe? Stop the simulation. Now change the velocity of the red puck to $-5.0\ \text{cm/s}$. Run the simulation again. What is the final velocity? What is the relative velocity of the pucks? Is momentum conserved in this collision? Is kinetic energy conserved in this collision?

4. Click the Magnetic Pucks button. Run the simulation. What happens to the path of a puck as it closely approaches the other puck? Allow the simulation to run for at least 90 second. Where are the pucks at this time? Why would this be their usual final position?

5. Consider the result of Item 5. Is there a way to set up the simulation that will prevent the pucks from ending up there? If so, what is it?

<u>b. Intermediate Level</u>

6. Reduce the velocity of the red puck to zero and move it to the upper right corner of the grid. Place the blue puck in left center of the grid. Set $v_{i,x} = 17\ \text{cm/s}$ and $v_{i,y} = 0$. Run the simulation. Pause the simulation after each of the first three collisions with the wall. Record the velocity of the puck before and after each of these collision. What fraction of its velocity remained after each collision? What fraction of its momentum remained? What fraction of its kinetic energy remained?

7. Is the collision between puck and wall elastic, inelastic, or partially elastic?

8. Drag the blue puck $14\ \text{cm}$ to the left of the origin (crosshairs) on the *x*-axis. Set its velocity to $10\ \text{cm/s}$. Drag the red puck $14\ \text{cm}$ to the right of the origin on the *x*-axis. Set its velocity to $-5\ \text{cm/s}$. Change the coefficient of restitution to $\varepsilon = 0.2$. Run the simulation. What is the final relative velocity of the pucks receding from each other? What is the ratio of their relative approaching velocity to their relative receding velocity? How does this compare to the value of ε that you chose?

<u>c. Advanced Level</u>

9. Set the velocity of the red puck to zero. Place the blue puck at a position of $y = -15\ \text{cm}$ with an initial velocity $v_{i,y} = 9\ \text{cm/s}$ in the positive *y*-direction. Tilt the table to an angle of $0.1°$. Run the simulation. When the blue puck reaches its maximum displacement, pause the simulation and measure this displacement. Using trigonometry, calculate how far above the ground the puck is and what its gravitational potential energy is at this moment. Start the simulation again and observe what the final velocity of the puck is when it reaches it original starting point. Was energy conserved?

10. Tilt the table at an angle of $0.1°$. Position the blue puck at $(-3,-4)$ and the red puck at $(8,-3)$. Set the initial velocity of the blue puck to $v_{i,x} = 2.0\,\text{cm/s}$ and $v_{i,y} = 5.0\,\text{cm/s}$, Set the initial velocity of the red puck to $v_{i,x} = -2.0\,\text{cm/s}$ and $v_{i,y} = 5.0\,\text{cm/s}$. Run the simulation for exactly $6.3\ \text{s}$. Slow down the simulation speed if you need to get the timing just right. Calculate the total initial and final energy of the system. Is energy conserved?

Lab Guide
Spring and Pendulum

I. Objectives

After completing this simulation, the student will be able to:
1. Describe the effect of mass on the period of a simple pendulum
2. Describe the effect of length on the period of a simple pendulum
3. Describe the effect of mass on the period of a mass-spring system
4. Describe the effect of spring constant on a mass-spring system
5. Describe the effect of the acceleration due to gravity on a simple pendulum
6. Compare and contrast frequency with period for pendula and mass-spring systems.

II. User Interface and Simulation Features

This simulation shows the periodic motion of both a simple pendulum and a hanging mass-spring system by plotting displacement versus time on a graph. You can vary the length and the mass of the pendulum. You can vary the mass and spring constant of the mass-spring system. Gravitational acceleration can be changed. The scale of the time axis on the graph can be changed from a full scale value of 5 seconds to that of 25 seconds. The bar graph on the left shows the displacement of the pendulum bob (in red). The bar graph on the right shows the displacement of he massive block in the mass-spring system. The small black arrow points to the approximate equilibrium position of the block.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Change the time axis to 25 seconds by “unchecking“ the time axis button. Run the simulation. Which has the greater period, the pendulum or the mass-spring system? Which has the greater frequency? What is the period of the mass-spring system? What is the period of the pendulum?

2. Change the time axis to 25 seconds. Change the mass of the pendulum to $2.0\ \mathrm{kg}$. Run the simulation. What is the period of the pendulum now? What effect does the mass have on the period of the pendulum?

3. Change the time axis to 25 seconds. Change the spring constant to $7.0\ \mathrm{N/m}$. Change the mass of the mass-spring system to $5\ \mathrm{kg}$. Run the simulation. Compare the periods and frequencies of the two systems. What is the meaning of your observation?

4. Change the time axis to 25 seconds. Change the length of the pendulum to $3.5\ \mathrm{m}$. Run the simulation. Compare the period of the pendulum to the value obtained in Item 1. What effect

does decreasing the length of the pendulum have on the period? What effect does it have on the frequency? Specifically, if you halve the length of the pendulum, what is the change in value of the period?

5. Change the time axis to 25 seconds. Change the mass of the mass-spring system to $0.1\ \mathrm{kg}$. Run the simulation. Compared to the result from Item 1, what happens to the frequency of the system? What happens to the period? What value do you obtain for the frequency in Hz?

b. Intermediate Level

6. What is the shortest pendulum period achievable in the simulation? How did you achieve it?

7. Change the time axis to 25 seconds. Set $k = 0.1\ \mathrm{N/m}$. What is the longest mass-spring period that you can reasonably measure using this setting? How did you achieve it? What part of a period did you measure? Why can you not reasonably measure a longer period?

8. Change the time axis to 25 seconds. Now set the mass of the mass-spring system to $10\ \mathrm{kg}$. Within the constraints of the simulation, what value for k do you need to achieve a value closest to the value for the period you found in Item 7? According to the theoretical equations, what value would you need?

9. If you needed to keep accurate time with one of these devices while moving from planet to planet, where $g \neq 9.81\mathrm{m/s^2}$, which one would you choose? Why would you make this choice?

c. Advanced Level

10. From a rigorous mathematical perspective, does a simple pendulum demonstrate simple harmonic motion? What about a mass-spring system?
11. Sometimes a mass-spring system is shown horizontally, with the mass sliding on a surface. Under what conditions would g affect the period and frequency of such a mass-spring system?

Lab Guide
Damped / Driven Oscillator

I. Objectives

After completing this simulation, the student will be able to:
1. Find and demonstrate the resonant frequency of a driven oscillator
2. Describe the effects of mass and spring constant on resonant frequency of an oscillator
3. Describe a damped oscillator in terms of drag coefficient and velocity.
4. Explain the "beat frequency" between driver and oscillating mass
5. Demonstrate an underdamped, critically damped and overdamped oscillator.

II. User Interface and Simulation Features

This simulates a mass on a spring that can be driven by an oscillator, damped by a viscous fluid, or both. The amplitudes of the oscillations of the mass and the oscillator are plotted versus time on separate graphs. The time axis can be changed from 5 s full scale to 50 s full scale. A pictorial model of the oscillating mass is accompanied by a bar graph indicating instantaneous displacement. You can adjust the drag coefficient, the mass of the block, the spring constant, and the oscillating driver frequency. The driving oscillator can be turned on or off. Note that the theoretical equations give angular frequency in rad/s, and the frequency of the oscillator in the experiment is given in Hz.

The digital values for mass, spring constant, and driver frequency used in calculations are controlled by your use of the analog sliders. These digital values may differ very slightly from the displayed values, which are rounded. It is possible to produce slightly different graph shapes depending on whether you are on the low end or high end of the displayed values. This is the visual equivalent of rounding a calculator value to the correct number of significant digits in a written exercise and does not indicate an error in your work.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Note that the oscillator is turned off. Run the simulation for 20 s. Pause the simulation. Print the graph. Describe the motion of the block?

2. Set the drag coefficient to 2.0. Run the simulation for 20 s. Pause the simulation. Print the graph. Compare and contrast the motion of the block with your observations in Item 1. What difference do you observe?

3. Turn the oscillator on. Run the simulation for 20 s. Pause the simulation. Print the graph. Compare and contrast the motion of the block with your observations in Item 1. Is there a significant difference in the motion?

4. Set the drag coefficient to zero. Turn the oscillator on. Run the simulation for 20 s. What is the difference in the motion of the block compared to Item 3?

b. Intermediate Level

5. Set the drag coefficient to zero. Turn the oscillator on. Run the simulation several times for 25 seconds each, reducing the driver frequency by 0.3 Hz each run. Do you find a value for the driver frequency that gives a maximum amplitude? What is the value? Does the block have a constant amplitude or does it change?

6. Set the drag coefficient to zero. Turn the oscillator on. Set the mass to $2.4\ \text{kg}$. Set the spring constant to $8.0\ \text{N/m}$. Note that these are twice the default settings. As in Item 5, try to find the resonant frequency for the new settings. What value can you set that is closest to the resonant frequency?

c. Advanced Level

7. Regardless of the driving frequency, the oscillator produces a wave in the mass-spring system. The block has the resonant frequency of $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. The two waveforms exhibit interference. Suppose the driving frequency is nearly, but not exactly, the resonant frequency. You can observe on the graph a "beat" frequency. This is like what you hear from two musical instruments playing the same note, but a bit out of tune. Set the drag coefficient to zero. Turn the oscillator on. Set the driving frequency to $0.4\ \text{Hz}$. Recall the resonant frequency from Item 5. Run the simulation for $50\ \text{s}$. Carefully observe the shape of the curve. How much time elapses between one minimum amplitude waveform and the next? This is the period of the beat frequency. Use the relationship $fT = 1$ to find the frequency f of the beat. Round the beat frequency to the nearest $0.1\ \text{Hz}$ (the closest that you can read on the slider bar). What is the difference between the driving frequency and the resonant frequency? How does this compare to the beat frequency?

8. Repeat Item 1. Then run the simulation several more times, increasing the drag coefficient b by an increment of $0.2\ \text{kg/s}$ for each run. How does the motion of the block change as you increase b?

9. If a system is underdamped with a small value of b, it will oscillate above and below zero amplitude. If a system is critically damped with an intermediate (but definite) value of b, it will not oscillate below zero amplitude. If a system has a value of b above the critical value, it is overdamped. The critical value is given by $b^2 - 4mk = 0$. Calculate the critical value for the drag coefficient b for the default settings of the simulation. Set the drag coefficient to this value and run the simulation. What do you observe? Stop the simulation. Set b to a value about $1.5\ \text{kg/s}$ less than the critical value. Run the simulation again. Carefully watch the displacement bar graph at the right of the screen. Now what do you observe?

Lab Guide
Basic Torque

I. Objectives

After completing this simulation, the student will be able to:
1. Define torque in terms of mass and distance.
2. Describe how torque changes with changes of mass and distance from axis of rotation
3. Demonstrate addition of torques by using multiple masses
4. Describe how moment of inertia changes with changes of mass and distance from axis of rotation
5. Demonstrate torque, moment of inertia and angular speed for a simple pendulum and a physical pendulum

II. User Interface and Simulation Features

This simulation consists of a rod to which one to four masses may be attached. Three of the four masses can have a mass from 1 to 10 kg. The red mass can have a mass of up to 12 kg. The rod itself may be made massless, or it can have a mass up to $4.0\ \mathrm{kg}$, selectable with slider bars. A bar graph display shows instantaneous values for angular position, speed and acceleration. When the simulation is run, torque around the axis causes the masses to revolve around the axis. During the simulation, a box displays the initial torque and the moment of inertia for the choices of mass and position.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Run the simulation. What is the mass of the red mass? How far is it from the axis of rotation (the pivot point)? At what point in each oscillation does it have its maximum angular speed? At what point does it have its maximum acceleration? What is the initial torque around the axis?

2. Move the red mass to $160\ \mathrm{cm}$ left of the axis. Run the simulation. What is the initial torque? What causes the force on the mass and the rod? Now move the red mass to $160\ \mathrm{cm}$ right of the axis. Run the simulation. What is the initial torque in this position? How is the direction of torque related to its sign?

3. Add the green mass and set its mass to $4.0\ \mathrm{kg}$. Run the simulation. What is the torque produced by the two masses? Now move the green mass to $120\ \mathrm{cm}$ to the right of the axis. Run the simulation. What do you observe? What is the initial torque of this system?

4. Run the simulation. What is the torque? Record this torque. Stop the simulation and set the number of masses to two, three and four. Run the simulation after each change and record the torque. Notice the side of the axis to which each mass is added. Now reset the simulation and

repeat setting up two, three and four masses. Run the simulation each time, recording the torque. What do you notice?

b. Intermediate Level

5. Using all four masses, what is the maximum positive torque you can produce? Does the rod contribute to this torque? Why?

6. Run the simulation. What is the moment of inertia? Now move the red mass to $120\ \mathrm{cm}$ right of the axis. Run the simulation again. Now what is the moment of inertia? Has it changed in sign? Why or why not?

7. Run the simulation. What is the approximate period of the oscillation? What is the torque? What is the moment of inertia? Now add a mass of $1.0\ \mathrm{kg}$ to each side of the rod at a distance of $160\ \mathrm{cm}$ from the axis. Run the simulation again. With this apparatus, can you have the same torque and a different period? Why?

c. Advanced Level

8. Set the mass of the rod to zero. Run the simulation three times, setting the distance of the red mass from the axis to $40\ \mathrm{cm}$, $80\ \mathrm{cm}$. and $160\ \mathrm{cm}$. What of inertia is reported for each run? Now reset the simulation. Run the simulation an additional three times, setting the red mass at $2.0\ \mathrm{kg}$, $4.0\ \mathrm{kg}$ and $8.0\ \mathrm{kg}$. What moment of inertia is reported for each run? Allowing for rounding, what is the relationship between distance from axis and moment of inertia? What is the relationship between mass and moment of inertia?

Lab Guide
Gravitational Orbits

I. Objectives

After completing this simulation, the student will be able to:

1. Demonstrate the gravitational orbit of a satellite
2. Demonstrate that gravitational orbits are ellipses
3. Explain escape velocity
4. Explain the perturbation of a satellite's orbit by another satellite
5. Demonstrate the effect of velocity on the shape of an orbit
6. Demonstrate the effect of mass on the shape of an orbit.

II. User Interface and Simulation Features

This versatile simulation has many features that allow you to combine different satellites around one or two suns. The available satellites are either in the solar system when you start or are in a box in the upper left corner. To use one of these "boxed" satellites, drag it out into the solar system to a desired location. If you click and hold on the sun or a planet, the simulation displays the initial velocity vector. Double-clicking on the sun or a planet opens a dialog box that allows you to change its position or velocity before running the simulation. You can show or hide the grid and the controls, turn on and off the velocity (blue) and acceleration (red) vectors, and turn on or off the "trails" of the satellites in their orbits by checking or unchecking the buttons. In the upper left corner of the screen is a magnification selector that can zoom in $(+)$ or out $(-)$ on the solar system. This is useful for satellites with very eccentric orbits or if you wish to check to see if a satellite appears to be escaping, or is just approaching aphelion.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Open the scenario "Single Planet." Run the simulation. The planet is Earth. What is the shape of the orbit? Double-click on the planet. What is the direction of its velocity relative to the sun's position? Does Earth begin at perihelion or aphelion? Stop the simulation. Now increase the x-velocity from $189\,\mathrm{m/s}$ to $275\,\mathrm{m/s}$. Run the simulation again. Now does the Earth begin at perihelion or aphelion? Does a satellite have a greater velocity at aphelion or perihelion?

2. Open the scenario "Single Planet." Can you find an initial x-velocity that gives a nearly circular orbit? What is its value?

3. Open the scenario "Two Suns." In this scenario, one sun is luminous and the other is a dark body of equal mass. Run the scenario with one planet. Describe the revolution of the two suns.

Now move the planet Earth to the coordinates $(-4,193)$. Move Neptune to the coordinates $(134,159)$ with a velocity of zero in both x and y. Run the simulation. How many passes past the two suns does Neptune make before it appears to escape their gravity? Experiment with fractional changes in the x-position. Do these have an effect on the shape of the orbit?

4. Open the scenario "Escape Velocity." Examine the initial velocities and distance from the sun of each planet. Run the simulation. Based on your observations, do any of the planets have enough initial velocity to escape the gravity of the sun? Which one? Why?

5. Open the scenario "Escape velocity." In this simulation, Earth is much closer to the sun than Mars. Experiment with the velocity of Earth. Approximately what is the escape velocity for Earth in this simulation?

6. Open the scenario "Two Planets." Run the simulation. Is there any visible interaction between the gravitational fields of the planets? Which would have more effect, doubling the mass of Venus or halving the distance between the two planets? Try to change the properties or location of Venus to demonstrate such an interaction.

<u>b. Intermediate Level</u>

7, Open the scenario "Opposite Motion." Run the simulation. Is there any physical reason that satellites cannot orbit the same body in opposite directions? Increase the masses of each of the two planets by a factor of 10. Run the simulation until at least $t = 50$ ..Is there any change in the orbits? Now increase the masses of each of the two planets by a factor of 100 times the original. Is there any change in the orbits? What do you observe about the sun? Why does this happen?

<u>c. Advanced Level</u>

8. Examine the interaction of three very massive bodies. Set the mass of Pluto to $m = 1000000\,\text{kg}$ at coordinates $(8,-147)$ with velocity $v = 189\,\text{m/s}$. Set the mass of Earth to $m = 500000\,\text{kg}$ at coordinates $(4,183)$ with velocity $v = -189\,\text{m/s}$. Run the simulation. What interaction do you observe between the bodies. Is it likely that this type of interaction can be accurately predicted much in advance?

Lab Guide
Cavendish Experiment

I. Objectives

After completing this simulation, the student will be able to:
1. Describe the Cavendish apparatus used to measure G , and the function of its components.
2. Describe the harmonic motion of the smaller dumbbell
3. Explain the effect of changing the mass of the stationary green dumbbell
4. Explain the effect of changing the spacing between the large masses and the small masses.
5. Calculate G from simulated laboratory data.

II. User Interface and Simulation Features

This simulation of the Mitchell-Cavendish apparatus and the Cavendish experiment shows two large fixed metal spheres attached to a beam. Two smaller spheres are attached to a second beam and are suspended from a thin fiber of known torsion constant. The centers of mass of the four spheres all lie in the same plane. The small spheres are deflected by the gravitational attraction of the large spheres. The small angular displacement is magnified by reflecting a laser beam from a mirror attached to the beam of the smaller spheres. The laser beam strikes a scale and its position is read. The masses of the two smaller spheres are identical and fixed. The masses of the two larger spheres are identical, but can be varied. The distance between the centers of mass of the large and small sphere can be changed. When the simulation is run, a strip chart records the position of the laser beam as it approaches equilibrium.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Run the simulation with the large masses at their default value of $1.5\ \mathrm{kg}$. Observe the oscillatory motion recorded on the display at the top of the screen. What is the period of oscillation? How long does the motion take to decay to equilibrium? What is the equilibrium displacement of the pointer?

2. Change the large mass to $0.5\ \mathrm{kg}$. Run the simulation. What is the period of oscillation? How long does the motion take to decay to equilibrium? What is the equilibrium displacement of the pointer?

3. Set the distance between centers of mass to $60\ \mathrm{mm}$. Run the simulation. What is the period of oscillation? How long does the motion take to decay to equilibrium? What is the equilibrium displacement of the pointer?

4. Consider the displacement of the beam of light from the laser. Is this value divided by the length of the beam (300 mm) equal to $\sin\theta_{eq}$? Remember the law of reflection!

b. Intermediate Level

5. Why is the torsion constant given in $\text{N}\cdot\text{m}$? What happened to θ?

6. Run the simulation. Calculate the gravitational attraction between one of the small balls and one of the large balls? What is this value?

c. Advanced Level

7. Is there a gravitational attraction between the small mass and the other, more distant large ball? Calculate the magnitude of this force. What error in torque do we incur if we neglect this correction? Remember that there are two large balls in the apparatus.

8. Run the simulation with default settings. Recall the length of one period of this torsion pendulum. Note that it takes nearly three hours for the apparatus to come to a stable equilibrium position. Why are these values so great? Calculate the moment of inertia I around the axis for the yellow dumbbell. What is its value? Angular acceleration is given by $\alpha = \tau / I$. What is the initial value of α for the default settings? But this is only at the moment of release of the masses. The value of α changes as torque changes. Going a step further, the period of a torsion balance is given by $T = 2\pi\sqrt{\frac{I}{\kappa}}$. Calculate the period for the default settings.

9. Run the simulation using the default settings. At equilibrium, the net torque is zero and $\frac{2dGMm}{r^2} - \kappa\theta_{eq} = 0$. Using the equilibrium values, evaluate G. What answer do you obtain? What percentage error do you calculate for this value versus the published value?

Lab Guide
Two Dimensional Oscillator

I. Objectives

After completing this simulation, the student will be able to:
1. Find the resonant frequency in both *x*- and *y*-directions
2. Calculate the resonant frequency in both *x*- and *y*-directions
3. Describe the effect of drag coefficient and experimentally find its critical value
4. Demonstrate the effect of different frequencies in the *x*- and *y*-directions.
5. Demonstrate the effect of initial velocity on the motion of the puck.

II. User Interface and Simulation Features

A puck is attached to the four midpoints of the sides of a square by four identical springs. When the puck is centered, the springs are all at their equilibrium point. They exert no net force on the puck. This means that when the puck is displaced, each individual spring tends to return to its equilibrium position. It is important to remember that there are four springs acting on the puck, each with its own spring constant and each with both an x-component and y-component of its force vector. One end of one horizontal spring is attached to a fixed side of the grid and does not move. The other horizontal spring is attached to an oscillating block. The same is true of the vertical springs.

You can drag the puck to a desired position. You can change the initial velocity by dragging the tip of the blue velocity vector. The x- and y-oscillators can be turned on or off independently. The spring constant, mass of the puck, drag coefficient and oscillator frequencies can be varied. The single value of the spring constant controls all four springs at once. Display of vectors and the path of the puck can be turned on and off. The blue vector represents velocity and the green vector represents acceleration. The vector length is proportional to the scalar term.

III. Questions

In each case when you start a simulation for the following questions, reset the values using the icon in the control panel at the left side of the screen. Each question assumes that you have done this. Then set only the values indicated in the question.

<u>a. Introductory Level</u>

1. Place the puck off-center on the x-axis. Set the initial velocity and the drag coefficient to zero. Run the simulation. Allow the puck to go through exactly four complete oscillations, pausing the simulation after the fourth. Divide the elapsed time by four to find the natural period of this system. What is the value of T? Calculate the frequency of this oscillation in Hz. What is the value of f? Stop the simulation. Set the horizontal oscillator frequency as close to this value as possible. Turn the horizontal oscillator on. Run the simulation. Describe the puck's motion.

2. Verify that the vertical springs are identical to the horizontal springs. Place the puck off-center on the y- axis. Set the initial velocity and the drag coefficient to zero. Run the simulation.

Calculate the natural period and frequency as you did in Item 1. Stop the simulation. Set the vertical oscillator frequency as close to this value as possible. Turn the vertical oscillator on. Run the simulation. Describe the puck's motion.

3. Calculate the resonant frequency for the settings used in Item 1. Use the values from Items 1 and 2 in the formula $f = \frac{1}{2\pi}\sqrt{\frac{4k}{m}} = \sqrt{\frac{k}{m\pi^2}}$. The factor of 4 under the radical is present because there are four springs exerting force on the puck.

4. Place the puck off-center on the x- axis. Set the initial velocity and the drag coefficient to zero. Set $k = 3.0 \text{ N/m}$ and $m = 1.0 \text{ kg}$. Start the horizontal oscillator frequency at 0.40 Hz. Run the simulation several times for at least 30 seconds each time. Increase the horizontal oscillator frequency by 0.05 Hz for each successive run. What is the value of the resonant frequency for these settings?

5. Move the puck horizontally so that it is on the y-axis. Set the initial velocity to zero. What is the default value for the drag coefficient? Run the simulation. Is the system underdamped, critically damped, or overdamped? Stop the simulation. Increase the drag coefficient. Is it possible to critically damp the system with the default values for k and m?

<u>b. Intermediate Level</u>

6. Move the puck horizontally so that it is on the y-axis. Set the initial velocity to zero. Set $m = 1.0 \text{ kg}$ and $k = 3.0 \text{ N/m}$. Increase the drag coefficient until you reach critical damping. What value of the drag coefficient provides critical damping?

7. Set the initial velocity and drag coefficient to zero. Set the horizontal oscillator to 0.25 Hz and turn it on. Set the vertical oscillator to 0.31 Hz and turn it on. Run the simulation. Describe the motion of the puck. If you were to run the simulation for a long time, say, five minutes, what do you think the shape of the motion would be? Experiment by running the simulation with a greater or lesser difference between the vertical and horizontal frequencies.

<u>c. Advanced Level</u>

8. Run the simulation. When the puck has nearly stopped, pause the simulation and click on the Plot Full Trails button. Describe the path of the puck.

9. Set the drag to zero. Run the simulation. Does the green acceleration vector point to the origin? Why is this? Now reset the simulation and run it again. Compare and contrast the direction of the green acceleration vector to the direction from the puck to the origin.